Growing through Vintages*

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Preliminary

Abstract

What drives long-run economic growth—the productivity of breakthrough ideas, or the rate at which they arrive? We address this question by studying the reallocation of research, entrepreneurship, and labor across finely defined industries, drawing on the universe of U.S. patents (1875–present), a newly digitized Census of Manufactures (1860–1940), and newly harmonized County Business Patterns (1946–present). We develop a semi-endogenous growth model in which researchers choose between creating product varieties and pioneering entirely new categories, which we call *nests*. Each new nest sets off a boom-and-bust cycle: varieties and industry value added rise rapidly following its birth, then decline as newer, more productive nests crowd it out. We estimate the model using maximum likelihood on 650 patent classes and validate it using data on value added, employment and establishment growth as well as a text-based measure of patent influence following Kelly, Papanikolaou, Seru, and Taddy (2021). Quantitatively, the higher productivity of new breakthrough ideas accounts for 83 percent of U.S. economic growth since the 1870s. The arrival of highly-productive nests in the estimated model is sufficiently rare to generate aggregate economic fluctuations.

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The 1920's were the decade of cars, the 1930's the decade of refrigerators, the 1940's that of pharmaceuticals (especially antibiotics), and the 1950's that of television, with telecommunications and computers taking over in recent decades.

— Harberger (1998)

1. Introduction

In his 1998 AEA presidential address, Arnold C. Harberger proposed two contrasting visions of economic growth. Yeast causes bread to rise evenly, whereas mushrooms pop up unevenly and unpredictably. Traditional growth models, he argued, resemble the yeast process, whereas actual growth patterns are heterogeneous and difficult to forecast, more like mushrooms. Since 1998, the new generation of growth models that emerged after Klette and Kortum (2004) incorporates mushroom-type mechanisms in which ideas arrive randomly across firms. With these models, economists have deepened their understanding of the heterogeneity in firm size, growth, and patenting activity, as well as their implications for aggregate economic growth.[1]

In the epigraph above, however, Harberger emphasizes industry concentration and the shifting of economic activity across industries from decade to decade, patterns that existing growth models do not capture. Continuing with biological metaphors, such growth resembles patches of berries in a field. Their emergence may be difficult to predict, but when they appear, multiple plants flourish together, producing similar fruits and competing for similar resources.

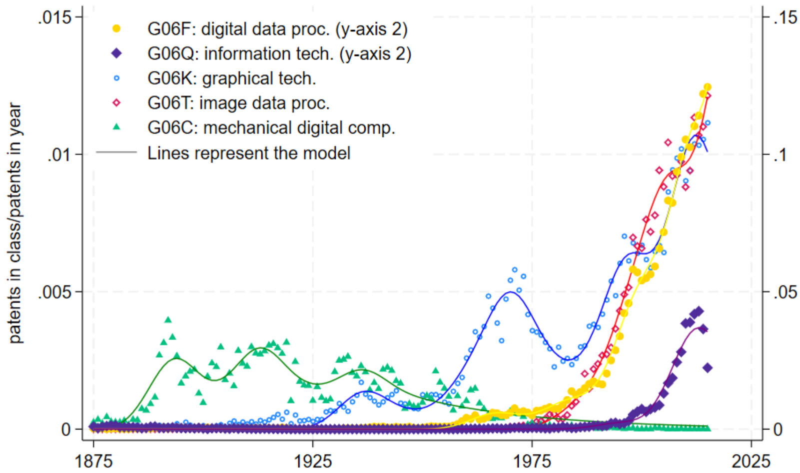
Empirically, we examine these industry cycles using historical data on the universe of U.S. patents (1875–present), a newly digitized Census of Manufactures (1860–1940), and newly harmonized County Business Patterns (1946–present).

Theoretically, we develop a model that features discontinuous innovation within industries, reminiscent of Schumpeter (1934) and Kuznets (1940). A major technological breakthrough attracts entrepreneurs, researchers, and labor into an industry. As competition intensifies and opportunities to improve on the original idea are exhausted, researchers and entrepreneurs redirect their efforts toward new breakthroughs elsewhere, generating new growth cycles in other industries.

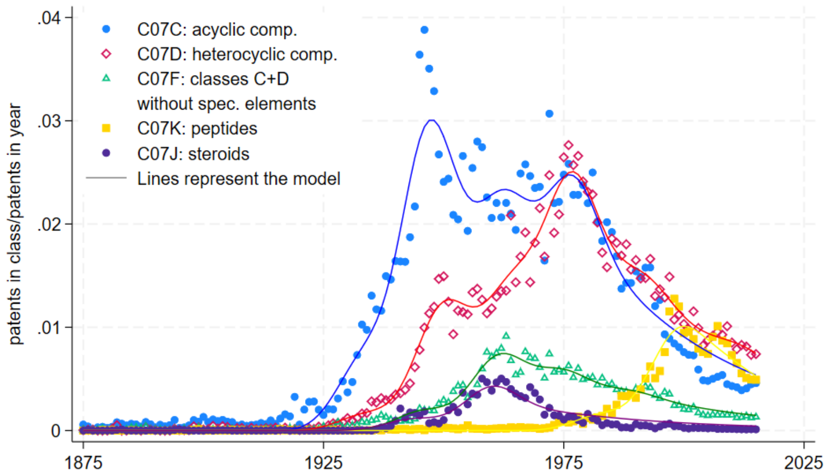
In the model, consumers have preferences over a continuum of varieties grouped into nests, with varieties more substitutable within nests than across them.[2] We interpret each nest as a distinct type of good, and the creation of a nest as a technological

[1]See Aghion, Akcigit, and Howitt (2014) for a survey of this literature.

[2]These preferences, introduced by Atkeson and Burstein (2008), are often used to study variable markups. See, for example, Eaton, Kortum, and Sotelo (2013), Edmond, Midrigan, and Xu (2015), and Amiti, Itskhoki, and Konings (2019). Here, markups are constant because there is a continuum of varieties in each nest.



(A) G06: Computing; Calculating or Counting



(B) C07: Organic Chemistry

FIGURE 1: Number of patents over time for top five classes in CPC codes G06 and C07

breakthrough. For example, we think of the telegraph, telephone, and smartphone as successive nests within telecommunications.

Researchers choose between creating new varieties within existing nests and creating entirely new nests. Varieties are sold in the market and the nest creator gets a share of the profits of the varieties in her nest. A nest's productivity is zero at its birth, rises rapidly, then declines as the nest ages. Varieties and employment in a nest follow a bell-shaped path—a boom as the breakthrough is exploited, and a bust as newer, more productive nests crowd it out. Nests share this common bell-shaped path but differ in when they are born and how high they peak. Aggregate growth comes from the expanding number of varieties and nests, as in Romer (1990), and from the rising productivity of successive nests, as in Aghion and Howitt (1992) and Grossman and Helpman (1990).

The model predicts that the patent share in a class is a finite sum of bell-shaped curves sharing a common shape but differing in birth dates and peak heights—as shown by the fitted lines in Figure 1. On average, we estimate 4.34 nests per patent class. With 8.7 parameters on average (2 parameters per nest $\times$ 4.34 nests) and 141 years of data (1875–2015) per class, the median R-squared is 0.80 and the mean is 0.75, suggesting that boom-and-bust cycles are a general feature of patent data, not merely as exemplified in Figure 1.

We scrape data from 2.8 million patents to construct a measure of patent influence a la Kelly et al. Out of sample, we confirm that both the birth of nests and their productivity are associated with the filing of the most influential patents, and of the patents that get the most citations.

We then gather the information on nests births and heights to recover the Poisson arrival rate of new nests and the log-normal distribution of nest productivity in the aggregate economy. With only a few parameters, the estimated model reproduces several empirical regularities: the number of active patent classes per year, the autocorrelation of patent counts within a class over time, and the cross-sectional distribution of patents across classes.

The estimates imply that the productivity of new nests rises over time, while the number of ideas per researcher declines, consistent with the notion in Jones (1995) that ideas are becoming harder to find. In all, growth in the productivity of new nests accounts for 83 percent of aggregate economic growth, growth in the number of nests accounts for 11 percent, and growth in varieties accounts for only 6 percent.

In short, the *quality* of new ideas — not their *quantity* — is the primary engine of long-run growth.

The Poisson arrival rate of nests is low — only three percent of patent classes receive a new nest in a typical year—and their heterogeneity is high—the 90-10 ratio of nest productivity is 22. As a result, the arrival of highly productive nests generates aggregate economic fluctuations. Simulating the model away from balanced growth produces boom-and-bust cycles with a standard deviation of income around trend of 0.02 and a duration of 40 to 50 years. The arrival of a few highly productive nests draw research toward exploiting them (through new varieties) and away from the creation of new nests. As a result, the economic boom caused by these nests is followed by an endogenous downturn. In particular, the large breakthroughs in the computing sector in the 1970s and 1980s, detected in the estimation, are associated with both the productivity boom in the late 1990s and the more recent productivity slowdown.

Related literature Our paper builds on the capital vintages literature, pioneered by Solow (1960) and Arrow (1962). To explain the protracted adoption of new technologies and the coexistence of new and old technologies documented by the historical literature, economists have emphasized learning by doing and sunk costs (Benhabib and Rustichini, 1991; Jovanovic and Lach, 1989; Chari and Hopenhayn, 1991; Jovanovic, 1998; Redding, 2002; Atkeson and Kehoe, 2007), and more recently, the gradual investment in innovation in new vintages (Ribeiro, 2024). This literature treats the arrival of new vintages as exogenous, typically one per period. Here, we introduce a tradeoff between investing in new varieties within existing vintages or investing in new vintages (nests), and a tradeoff between investing in better or in more vintages. For simplicity, we make exogenous technology growth within vintages studied in the literature. Our estimation procedure provides a novel method of identifying the birth of new vintages in patent data and testing for the number of vintages per patent class.

If we interpret the entry of firms as new vintages in Aghion, Bergeaud, Boppart, and Brouillette (2025), then their model features endogenous innovation within vintages, spillovers of knowledge across vintages (like Ribeiro, 2024) and adds a free entry condition for new vintages (firms). Aghion, Bergeaud, Boppart, and Brouillette (2025) estimate that the growth in quality across vintages accounts for 83 percent of aggregate growth, while growth in variety accounts for 17 percent, almost exactly our estimates.[3] In the model, the creation of new nests features both knowledge externalities and the creative destruction of older nests, the two mechanisms emphasized by Caballero and Jaffe (1993). Using data on market value and patents, Caballero and Jaffe (1993) estimate that creative destruction is about 4 percent per year, again very close to the 3 percent rate of nest obsolescence that we estimate.

The arrival of a new vintage in the model leads to a discontinuous burst in the creation of varieties within industries. We do not observe firm boundaries and the model does not take a stance on how varieties are distributed across firms. A promising path for future work is to reconcile these bursts with the bell-shaped evolution in the number of firms following the introduction of a new product class in Gort and Klepper (1982), Utterback and Suárez (1993), Klepper and Miller (1995), Klepper (1997), as well as with bursts in the creation of new products within firms in Berlingieri, De Ridder, Lashkari, and Rigo (2025).[4]

[3]The 0.17 here further decomposes into growth in the number of nests (0.11) and of varieties (0.06). Most growth occurs across technology classes (nests) in our estimation because patent classes that were large 130 years ago are unrelated to those that are large today (see Figure 3(B) and Figure 10(A) for industries). These estimates are not necessarily inconsistent with Lentz and Mortensen (2008) and Garcia-Macia, Hsieh, and Klenow (2019)'s findings that most growth is generated by incumbent firms. We do not take a stance on firm boundaries, and the time horizon in these papers is much shorter.

Also related are Argente, Lee, and Moreira (2024)'s study the life cycle of products at the barcode level, Teng (2025)'s study of the composition of research teams over the firm lifecycle, and Fieler and Eaton (2025)'s decomposition of growth using trade data.

[4]Aghion, Akcigit, Bergeaud, Blundell, and Hémous (2019) associate increases in top income inequality to breakthrough innovations. Varieties in our model all have the same profits, as in Klette and Kortum (2004), and so introducing firms and markups as in Peters (2020) may be feasible.

Consistent with the entry of firms, in Section 6.1.3, we decompose value added growth into growth in

The distinction between innovating in nests (which are not sold) and varieties (which are sold) parallels the distinction between invention and adoption in Arkolakis, Ramondo, Rodríguez-Clare, and Yeaple (2018) and Trouvain (2024), as well as the distinction between basic and applied research in Nelson (1959), Cheng and Dinopoulos (1996), Akcigit and Kerr (2018), and Akcigit, Hanley, and Serrano-Velarde (2021).[5]

Finally, the paper relates to the literature on structural transformation introduced by Kuznets (1966). To capture permanent shifts in the economy from agriculture to manufacturing and later to services, economists have developed models with unbalanced growth (Acemoglu and Guerrieri, 2008; Herrendorf, Rogerson, and Valentinyi, 2013; Duarte and Restuccia, 2010; Comin, Lashkari, and Mestieri, 2021).[6] In contrast, we study waves of innovation and ensuing shifts across narrower industries along a balanced growth path.[7]

The paper is organized as follows. Section 2 describes the data sources and empirical regularities. We present the model in Section 3 and solve for the balanced growth path in Section 4. In Section 5, we estimate the model using patent data. Section 6 conducts out of sample analysis. It includes non-targeted moments from the patent data, moments on value added, employment and number of establishments, as well as the textual analysis of patents. In Section 7, we simulate the model out of the balanced growth path to study aggregate fluctuations.

2. Data

We describe the sources of data in Section 2.1 and salient features of the data in Section 2.2.

2.1 Data: Sources

Patent Data We use information on the number of patents and on the text of patents from Google Patents. The data comprise all U.S. patents from 1875 to 2015.[8] Patent data are classified according to the Cooperative Patent Classification (CPC) into four-digit

the number of establishments (0.6) and in the value added per establishment (0.4), and show how to reconcile the model to these shares.

[5] Basic and applied research differ quantitatively in these models. Here, they differ qualitatively through the higher elasticity of substitution between varieties within nests than across nests.

[6] See Herrendorf, Rogerson, and Valentinyi (2014) for a survey; Buera, Kaboski, Mestieri, and O'Connor (2020) generalizes the concept of balanced growth to a stable transformation path.

[7] Grossman and Helpman (1991) feature a continuum of sectors with a quality ladder, in which growth shifts across sectors in discrete jumps rather than gradually, as in the vintage literature.

[8] The data, in `https://patents.google.com/`, are available since 1836 but we focus on the period since the Second Industrial Revolution when innovation shifted from individual tinkers to systematic and strategic R&D investments by firms (Mokyr (1992)). Growth in the number of patents is roughly linear in the period of our data and non-linear before. We also drop the most recent years, because we observe a sharp drop in the number of patents granted, and we don't know whether the drop is real or due to the time of compiling the data.

codes, which we call classes. We drop CPC codes with fewer than 100 patents and are left with 650 classes.

Census of Manufactures We use the Census of Manufactures digitized by Arkolakis, Lee, and Peters (2026). These data record the number of establishments, value added, and employment by industry and location (county or city) at decadal frequency from 1860 to 1940. Industries are classified with the 1950 classification used by the Integrated Public Use Microdata Series (IPUMS). Merging across decades gives us an unbalanced panel of 51 industries, and of 47 industries after merging with patent data.[9]

County Business Patterns (CBP) The CBP provide annual information on the number of establishments and workers of all U.S. sectors (not only manufacturing) and counties from 1946 to the present. We extend the work of Eckert, Fort, Schott, and Yang (2021a), which covers 1975–2018, to 1946. The 1946–1961 data use the SIC1945 and SIC1949 classifications, for which no machine-readable concordance to NAICS was hitherto available. We construct the two concordances to close this gap, making available data on the postwar peak of American manufacturing (1946–1961) at the four-digit level.[10] There are more than 200 industries in most years. This fine resolution is essential for our purposes since coarser industry aggregations mask within-industry reshuffling of labor across vintages.

Together, the Census of Manufactures and CBP provide a continuous record of establishment and employment dynamics spanning over 150 years. While we use only industry-year information here, our digitization and harmonization efforts make data on location also available.[11]

2.2 Empirical Regularities

Beyond the examples in Figure 1, we document several empirical regularities in the patent data that point to shifts in innovation across patent classes. Figure 2 plots the number of classes with at least one patent per year. Of the 650 patent classes in the data, the number of active classes has risen from roughly 450 in 1875 to 625 in 2015. Newer classes have emerged over time, while some old ones have become inactive.

[9] 46 industries appear in all years and 377 "aircraft and parts" appears only in 1929 and 1939.

[10] Our work completes the cross-walks in Eckert, Lam, Mian, Muller, Schwalb, and Sufi (2022) for 1946–1974. See Section E.1.2 for details on the harmonization procedure.

[11] The CBP also has information on the distribution of establishment sizes.

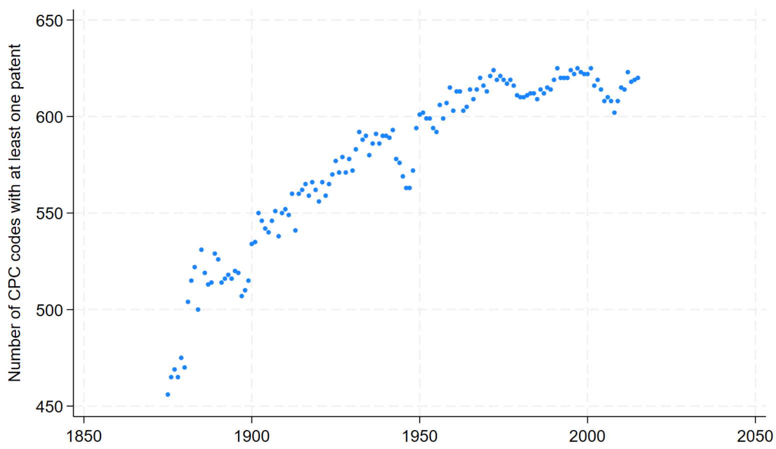
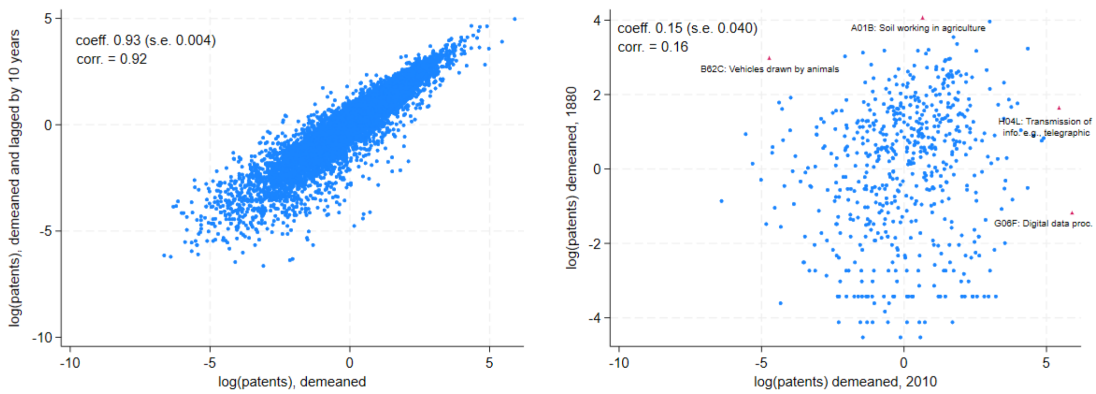


FIGURE 2: Number of CPC codes with at least one patent per year

Figure 3 plots the log of the number of patents in a cross-section, demeaned, against its lagged value, where the lag is 10 years in panel (A) and 130 years in panel (B). In general, classes with many patents today had many patents ten years ago, but patent counts today are uncorrelated with patent counts 130 years ago.



(A) Lagged 10 years

(B) Lagged 130 years

FIGURE 3: Number of patents (in logs and demeaned) and its lag

Note: For each CPC patent class, we compute the average number of patents over the five years centered on each decade (e.g., 2008–2012 for 2010), take logs, and demean. Panel (A) plots this demeaned log average against its value ten years prior, with one observation per CPC class per decade from 1890 onward. Panel (B) plots the same variable against its value 130 years prior, with one observation per CPC class.

Figure 4 plots the kernel density of the number of patents in logs and demeaned across classes for every two decades. Despite the reshuffling in the size of patent classes in Figure 3, the cross-sectional distribution of patent counts is stable over time.

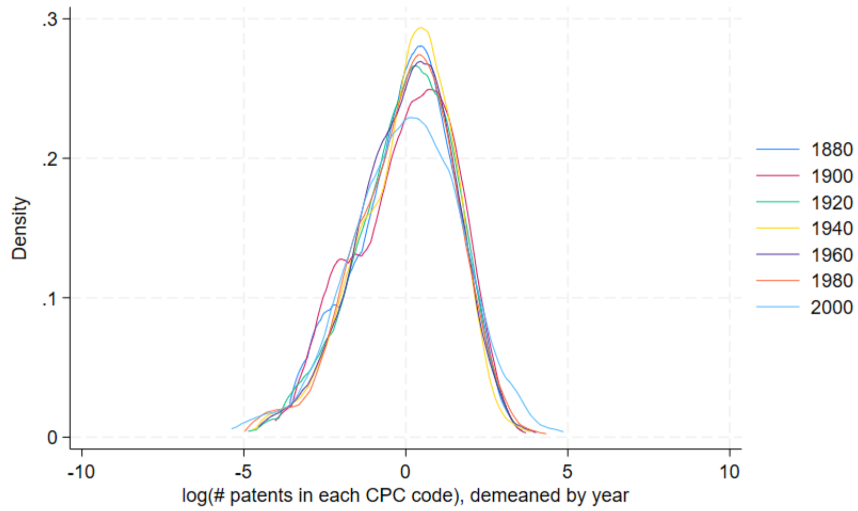


FIGURE 4: Kernel density of the number of patents across CPC codes for selected years

TABLE 1: Outcome growth and number of patents

Dependent variable: log change in the column outcome (yearly average)	Census of Mfg.	CBP		
	(1) Value added	(2) Employment	(3) Payroll	(4) Establishments
log (average number of patents per year)	0.056^{***}	0.016^{***}	0.021^{***}	0.015^{***}
	(0.016)	(0.004)	(0.006)	(0.004)
log change in number of patents (yearly avg.)	0.117	-0.010	-0.013	-0.024
	(0.166)	(0.043)	(0.065)	(0.047)
lagged log outcome	-0.082^{***}	-0.058^{***}	-0.067^{***}	-0.075^{***}
	(0.0071)	(0.0026)	(0.0045)	(0.0030)
Observations	263	2,477	1,458	2,489
R-squared	0.641	0.338	0.351	0.349
Industry FE	47	253	252	253
Year FE	6	13	7	13

Note: Standard errors in parentheses. The table presents the results of regression (1). All columns include industry and year fixed effects absorbed (`areg`). Column (1) uses Census of Manufacture data (dependent variable: log change in value added). Columns (2)–(4) use County Business Patterns (CBP) data on a 5-year Census (2/7) grid extended to 1947 (1952 and 1957, absent in CBP, are proxied by 1953 and 1959); employment and establishments cover 1947–2012, payroll only the post-1975 benchmarks. $^{***}\, p < 0.01$, $^{**}\, p < 0.05$, $^{*}\, p < 0.1$.

Patents and economic growth Table 1 associates the level of patenting activity in an industry with subsequent economic growth. We regress

$$(1)\quad \Delta y_{it,t-\ell} = \delta_1 \log\left(\overline{\#\text{ patents}}_{it,t-\ell}\right) + \delta_2 \Delta \log\left(\#\text{ patents}_{it,t-\ell}\right) + y_{it-\ell} + \delta_i^I + \delta_t^T + \epsilon_{it}$$

where $\Delta y_{it,t-\ell}$ is the annual growth between periods $t-\ell$ and t in a measure of economic activity in industry i (e.g., value added, employment). We regress it on the average number of patents per year between $t-\ell$ and t and in the average yearly growth in patents. We control for the initial level of economic activity $y_{it-\ell}$, industry fixed effects δ_i^I and time fixed effects δ_t^T. In the model below, economic growth is associated with an increase in the number of patents, but not their growth. That is, $\delta_1 > 0$ and $\delta_2 = 0$.

In column (1), we use data from the Census of Manufactures, a time lag ℓ of ten years (the time between censuses), and y_{it} is value added. In columns (2) through (4), we use the CBP data and set ℓ to five years. Because value added is not reported in the CBP, we use the number of workers, total payroll and number of establishments. The coefficient δ_1 is positive and significant in all cases, while δ_2 is always insignificant. For example, a point estimate $\delta_1 = 0.056$ in column (1) implies that one standard deviation in the log of the number of patents per year, 1.87, increases value added in an industry by 0.10 log points, three-quarters of the standard deviation in value added growth (0.14 log points).[12] The negative coefficient on initial value added indicates convergence.

The model in Section 3 is designed to capture the boom-and-bust cycles of innovation in Figure 1, the shift in innovation across types of goods in Figure 3 and their relationship to industry growth in Table 1. Over time, the range of goods expands, as shown in Figure 2. In the aggregate, the model features a balanced growth path with a stable distribution of economic activity across industries and patent classes, as shown in Figure 4.

3. THE MODEL

Time is continuous and infinite. There is an endogenous set of varieties divided into market segments called nests. Varieties are symmetric within nests. Worker households inelastically supply their endowment of labor and have identical preferences represented by a nested constant elasticity of substitution (CES) demand system.

There are two types of labor. Production labor $L(t)$ is the sole input into production. Research labor $H(t)$ may be allocated to the invention of new nests or new varieties. Researchers of varieties choose which nest to enter, and researchers of nests choose stochastically between a higher nest arrival rate or higher nest productivity. All labor markets are competitive.

[12] The results in Table 1 parallel those in Benhabib and Spiegel (2005). They find that the level of human capital, not its growth, is associated with GDP growth in cross country regressions. Fort, Goldschlag, Liang, Schott, and Zolas (2025) run regressions similar to Table 1 at the firm level. Because patenting and growth decisions are made within the firm, our regression remains closer in spirit to the cross-industry approach of Benhabib and Spiegel (2005)

3.1 Demand

Let $\Omega(t)$ denote the set of nests at time t, and ω an element of $\Omega(t)$. A representative household at time 0 has utility

$$U = \int_0^\infty e^{-\rho t} \log \left(\frac{\bar{C}(t)}{L(t) + H(t)} \right) dt \tag{2}$$

where

$$\bar{C}(t) = \left[\int_{\Omega(t)} C(\omega, t)^{1/\eta} d\omega \right]^\eta$$
$$C(\omega, t) = \left[\int_{\tilde{\omega} \in \omega} c(\tilde{\omega}, t)^{1/\sigma} d\tilde{\omega} \right]^\sigma$$

and where ρ is the discount rate, and $c(\tilde{\omega}, t)$ is the quantity of each variety $\tilde{\omega}$ in nest ω at time t. The elasticity of substitution between nests is $\eta/(\eta - 1)$ and the elasticity of substitution between varieties is $\sigma/(\sigma - 1)$. Assume $\eta > \sigma > 1$ so that the varieties are more substitutable within nests than across nests. The household can invest in a risk free bond with nominal interest rate $r(t)$.

3.2 Production and innovation

Production technology A nest ω is characterized by a productivity parameter $\bar{z}(\omega)$ and a time of birth $\tau(\omega)$, and we henceforth drop the notation ω refer to a nest with $(\bar{z}, \tau)$. The productivity of all varieties in nest $(\bar{z}, \tau)$ at time $t \geq \tau$ is

$$z(\bar{z}, \tau, t) = \bar{z}\psi(t - \tau). \tag{3}$$

Assumption 1. *(i) Function ψ is differentiable, increasing, and $\lim_{x \to \infty} \psi'(x)/\psi(x) = 0$. In addition, $\psi(0)$ is strictly increasing and $\psi(0) = 0$. (ii) $\lim_{x \to 0} \psi'(x)/\psi(x) = \infty$ and $\psi'(x)/\psi(x)$ is decreasing.*

These features are intended to capture learning about new technologies and the difficulties of developing old technologies as in the vintage models of Bloom, Jones, Van Reenen, and Webb (2020), Ribeiro (2024) and Aghion, Bergeaud, Boppart, and Brouillette (2025). Mechanically here, like in Antràs (2023), time itself improves productivity without explicit payments to labor or production. We impose Assumption A1(i) to ensure the existence of the balanced growth path and A1(ii) to further characterize the balanced growth path. Our parametrization of ψ in the estimation below satisfies both A1(i) and (ii), and $\psi \in [0, 1)$ so that $\bar{z}$ is the limit productivity in the nest as it ages, $\lim_{t \to \infty} z(\bar{z}, \tau, t) = \bar{z}$.

Creation of varieties Varieties face an exogenous death rate δ. Creating a new variety costs f_1 units of research labor. The measure of varieties in nest $(\bar{z}, \tau)$ at time t, denoted with $n(\bar{z}, \tau, t)$, evolves according to:

$$\dot{n}(\bar{z}, \tau, t) = -\delta n(\bar{z}, \tau, t) + \frac{h_1(\bar{z}, \tau, t)}{f_1}. \tag{4}$$

where $h_1(\bar{z}, \tau, t)$ is the measure of researchers, and throughout, $\dot{x}$ denotes the derivative of variable x with respect to time.

Creation of nests Researchers of new nests trade off the arrival rate against the productivity of their nests. For each researcher, nests arrive with a Poisson rate m and its productivity $\bar{z}$ is distributed log normally with standard deviation parameter θ and mean parameter $\log(\mathbf{z})$. Let $F(\bar{z}; \mathbf{z})$ denote the cumulative distribution function of this distribution and let $V(\bar{z}, t)$ denote the value of a nest with productivity $\bar{z}$ at time t (below). There is perfect risk sharing among researchers of new nests. The researcher chooses m and $\mathbf{z}$ to maximize the expected value of innovation:

$$\mathcal{V}(t) = \max_{m, \mathbf{z}} m \int_0^\infty V(\bar{z}, t) dF(\bar{z}; \mathbf{z}) \tag{5}$$

subject to the following CES technology constraint:

$$f_2 \left\{ \left[\bar{H}_2(t)^{-\nu_m} m\right]^{1/\kappa} + \left[\bar{H}_2(t)^{-\nu_z} \mathbf{z}\right]^{1/\kappa} \right\}^\kappa \leq 1 \tag{6}$$

where

$$\bar{H}_2(t) = \int_{-\infty}^t e^{\phi(s-t)} H_2(s) ds \tag{7}$$

and where $f_2 > 0$, κ, ν_m, ν_z, and ϕ are parameters, and $H_2(t)$ is the measure of researchers of nests at time t. The term $\bar{H}_2(t)$ captures the cumulative knowledge of researchers of nests at time t with a depreciation ϕ of past researchers. There are potential spillovers (positive or negative) of $\bar{H}_2$ on the efficiency of each individual researcher in producing more (ν_m) or better ideas (ν_z). The flow of nests at time t is $N(t) = H_2(t) m(t)$ where $m(t)$ solves (5).

3.3 Equilibrium

Household problem We take the aggregate consumption $\bar{C}(t)$ to be the numeraire and denote with $B(t)$ the stock of bonds. The household instantaneous budget is:

$$\dot{B}(t) = r(t) B(t) + Y(t) - \bar{C}(t) \tag{8}$$

where, with symmetric varieties, the cost of $\bar{C}(t)$ and income $Y(t)$ are

$$\bar{C}(t) = \int_{-\infty}^{t} N(\tau) \int_{0}^{\infty} p(\bar{z},\tau,t)c(\bar{z},\tau,t)dF(\bar{z};\mathbf{z}(\tau))d\tau.$$
$$Y(t) = L(t)w(t) + \int_{-\infty}^{t} N(\tau) \int_{0}^{\infty} n(\bar{z},\tau,t)\pi(\bar{z},\tau,t)dF(\bar{z};\mathbf{z}(\tau))d\tau$$

and where $p(\bar{z},\tau,t)$, $c(\bar{z},\tau,t)$, and $\pi(\bar{z},\tau,t)$ are respectively the price, quantity, and profit of each variety in nest $(\bar{z},\tau)$, and $n(\bar{z},\tau,t)$ is their measure.

Maximizing (2) subject to (8) and a no-Ponzi condition yields the allocation:

$$\begin{aligned} \dot{\bar{C}}(t) &= [r(t) - \rho]\bar{C}(t) \\ C(\bar{z},\tau,t) &= P(\bar{z},\tau,t)^{-\eta/(\eta-1)}\bar{C}(t) \\ c(\bar{z},\tau,t) &= n(\bar{z},\tau,t)^{-\sigma}C(\bar{z},\tau,t) \end{aligned} \tag{9}$$

where

$$P(\bar{z},\tau,t) = n(\bar{z},\tau,t)^{(1-\sigma)}p(\bar{z},\tau,t). \tag{10}$$

Firm and research problem At time t, the price of the varieties in nest $(\bar{z},\tau)$ is

$$p(\bar{z},\tau,t) = \frac{\sigma w(t)}{\bar{z}\psi(t-\tau)}, \tag{11}$$

the markup σ times marginal cost, where $w(t)$ is the wage of production workers. Using (9), (10) and (11), the profit flow is:

$$\pi(\bar{z},\tau,t) = \Upsilon_0\bar{C}(t)\left[\frac{\bar{z}\psi(t-\tau)}{w(t)}n(\bar{z},\tau,t)^{(\sigma-\eta)}\right]^{1/(\eta-1)} \tag{12}$$

where $\Upsilon_0 = (\sigma-1)\sigma^{-\eta/(\eta-1)}$. Here and below, we use Υ to denote constants. The creator of a variety gets a share $(1-\chi)$ of this flow profit and pays a share χ to the creator of the nest. Then, the value of a variety in nest $(\bar{z},\tau)$ at time t is:

$$v(\bar{z},\tau,t) = (1-\chi)\int_{t}^{\infty} e^{-\delta(s-t)-\int_t^s r(s')ds'}\pi(\bar{z},\tau,s)ds. \tag{13}$$

The value of this nest at its birth τ is:

$$V(\bar{z},\tau) = \chi\int_{\tau}^{\infty} e^{-\int_\tau^t r(s)ds}n(\bar{z},\tau,t)\pi(\bar{z},\tau,t)dt. \tag{14}$$

The value of the nest increases with $\bar{z}$ potentially through a higher profit per variety (term $\pi(\bar{z},\tau,t)$) and a greater measure of varieties (term $n(\bar{z},\tau,t)$).

Equilibrium Past research effort endows the economy at time 0 with a measure of nests $N(\tau)$, a distribution of productivity parameters $F(\bar{z};\mathbf{z}(\tau))$ for all $\tau < 0$, and measures of varieties $n(\bar{z},\tau,0)$ for all $\bar{z} \in \mathbb{R}_{++}$ and $\tau < 0$. We assume a zero net supply of bonds, $B(t) = 0$, in all periods. An *equilibrium at time 0* consists of researcher allocations $h_1(\bar{z},\tau,t)$ and $H_2(t)$, nest researcher choices $m(t)$ and $\mathbf{z}(t)$, consumption allocations $\bar{C}(t)$, $C(\bar{z},\tau,t)$, and $c(\bar{z},\tau,t)$, interest rates $r(t)$, production worker wages $w(t)$, and researcher wages $w_H(t)$, defined for all $t > 0$, $\tau \le t$, and $\bar{z} \in \mathbb{R}_{++}$ such that the following conditions hold.

1. The value of research in (5) and (13) satisfy $v(\bar{z},\tau,t) \le f_1 w_H(t)$ with equality whenever $h_1(\bar{z},\tau,t) > 0$, and $\mathcal{V}(t) \le w_H(t)$ with equality whenever $H_2(t) > 0$. Choices $m(t)$ and $\mathbf{z}(t)$ solve problem (5).
2. Household spending across time, nests and varieties satisfies (9), with the price $p(\bar{z},\tau,t)$ given in (11), and a net demand for bonds $B(t) = 0$.
3. The markets for production and research labor clear:

$$L(t) = \bar{C}(t)/[\sigma w(t)] \tag{15}$$

$$H(t) = H_2(t) + \int_{-\infty}^{t} N(\tau) \left[\int_0^{\infty} h_1(\bar{z},\tau,t) dF(\bar{z};\mathbf{z}(\tau)) \right] d\tau \tag{16}$$

4. Balanced Growth Path

In Section 4.1, we solve the problem of the researchers of nests. Its solution gives us the evolution of productivity and measures of new nests, and the evolution of varieties within nests as functions of the research allocation. We then derive this research allocation in Section 4.2. We solve the social planner's problem in Section 4.3.

4.1 Dynamics in the balanced growth path

All aggregate variables grow at constant rates. Denote the growth rate of variable x with g_x. The growth rates of population g_H and g_L are positive and exogenous. From (9), the interest rate is $r = \rho + g_{\bar{C}}$. We conjecture that the allocation of researchers between the creation of new varieties and new nests is constant, and that there is entry into all existing nests in all periods. Then, the profit flow may depend on time but not on the age or the productivity of a nest.

Researchers of nests Researchers of nests anticipate the measure of varieties, $n(\bar{z},\tau,t)$, when choosing the log-normal mean parameter $\mathbf{z}$. Inventors of varieties are indifferent between any of the existing nests. Setting $\pi(\bar{z},\tau,t) = \pi(\mathbf{z}(\tau),\tau,t)$ in (12), we get

$$n(\bar{z},\tau,t) = n(\mathbf{z}(\tau),\tau,t)\left(\frac{\bar{z}}{\mathbf{z}(\tau)}\right)^{1/(\eta-\sigma)}.$$

Substituting into the profit flow (12) and then into the value function (14), we obtain:

$$V(\bar{z},\tau) = V(\mathbf{z}(\tau),\tau)\left(\frac{\bar{z}}{\mathbf{z}(\tau)}\right)^{1/(\eta-\sigma)}$$

The problem of the researcher in (5) becomes:

$$\mathcal{V}(\tau) = \max_{m,\mathbf{z}}\left[\Upsilon_1 V(\mathbf{z}(\tau),\tau)\mathbf{z}(\tau)^{1/(\sigma-\eta)}\right] m\mathbf{z}^{1/(\eta-\sigma)}$$

subject to (6), where $\Upsilon_1 = \exp\{[\theta/(\eta-\sigma)]^2\}$ and where the researcher takes the term in square brackets as given. Assuming $\kappa \in (0,\min\{1,\eta-\sigma\})$, the solution is[13]

$$\mathbf{z}(\tau) = \bar{H}_2(\tau)^{\nu_z}\tilde{f}_2^{-1}(\eta-\sigma)^{-\kappa} \tag{17}$$

$$m(\tau) = \bar{H}_2(\tau)^{\nu_m}\tilde{f}_2^{-1} \tag{18}$$

where $\bar{H}_2(t) = H_2(t)/(g_H+\phi)$ and $\tilde{f}_2 = f_2[(1+\eta-\sigma)/(\eta-\sigma)]^{\kappa}$. The measure of nests invented at time τ is

$$N(\tau) = H_2(\tau)^{1+\nu_m}(g_H+\phi)^{-\nu_m}\tilde{f}_2^{-1}. \tag{19}$$

The parameter estimates below satisfy $\nu_m \in (-1,0)$ and $\nu_z > 0$. They imply that the measure of new nests $N(t)$ and their productivity parameter $\mathbf{z}(t)$ increase with $H_2(t)$, but the number of nests per researcher $m(t)$ decreases as in Jones (1995). We impose the following weaker assumption A2:

Assumption 2. $1+\nu_m > 0$ *and* $1+\nu_m+\nu_z/(\eta-\sigma) > 0$

Following Melitz (2003), we introduce $Z(t)$ as the technology of a typical nest at time t:

$$Z(t) = \left\{\frac{\int_{-\infty}^{t} N(\tau)\left[\int_0^{\infty}[\bar{z}\psi(t-\tau)]^{1/(\eta-\sigma)}dF(\bar{z};\mathbf{z}(\tau))\right]d\tau}{\int_{-\infty}^{t} N(\tau)d\tau}\right\}^{\eta-\sigma} = (\Upsilon_2)^{\eta-\sigma}\mathbf{z}(t) \tag{20}$$

where we use (17) and (19) for the second equality and where

$$\Upsilon_2 = \Upsilon_1 g_H(1+\nu_m)\int_0^{\infty} e^{-g_H[1+\nu_m+\nu_z/(\eta-\sigma)]s}\psi(s)^{1/(\eta-\sigma)}ds.$$

[13]See Appendix A.1 for the second order conditions.

This integral exists by Assumptions A1(i) and A2. If these assumptions fail, then the utility from past nests is greater than that of recent ones, and $Z(t)$ becomes unbounded.

Evolution varieties. Imposing equal profits across nests $(\bar{z}, \tau)$ in (12), we can write the evolution of varieties in a nest as:

$$n(\bar{z}, \tau, t) = \bar{n}(t) \left[\frac{\bar{z}\psi(t-\tau)}{Z(t)} \right]^{1/(\eta-\sigma)} \tag{21}$$

where $\bar{n}(t)$ is the average number of varieties per nest. Denote with $\tilde{g}$ the rate at which nests become obsolete in (21):

$$\tilde{g} = g_{\mathbf{z}}/(\eta - \sigma) - g_{\bar{n}} = g_H \left(\nu_m + \frac{\nu_z}{\eta - \sigma} \right)$$

The growth rates in the second equality are from (17) and (19) with $g_{\bar{n}} = g_H - g_N$. By Assumption A1(i), the nest is born with no varieties, $n(\bar{z}, \tau, \tau) = 0$, since $\psi(0) = 0$. Function ψ has decreasing growth rates with $\lim_{t\to 0} \dot{\psi}(t)/\psi(t) = \infty$ and $\lim_{t\to\infty} \dot{\psi}(t)/\psi(t) = 0$ by A1(i) and (ii). Then, variety growth is infinite at birth and monotonically declines to $-\tilde{g}$ as the nest ages. If $\tilde{g} > 0$ as in the parameter estimates, the evolution of nests is bell shaped.

Taking the derivative of (21) with respect to time and substituting into (4), research labor in nest $(\bar{z}, \tau)$ is

$$h_1(\bar{z}, \tau, t) = f_1 n(\bar{z}, \tau, t) \left[\delta - \tilde{g} + \frac{1}{\eta - \sigma} \frac{\dot{\psi}(t-\tau)}{\psi(t-\tau)} \right] \tag{22}$$

Since the last term converges to zero as the nest ages, the death rate must be higher than the obsolescence rate to ensure entry into all existing nests, $\delta > \tilde{g}$:

Assumption 3. $\delta > g_H(\nu_m + \frac{\nu_z}{\eta - \sigma})$.

Income The numeraire is the price of the aggregate good $\bar{C}(t)$:

$$1 = \int_{-\infty}^{t} N(\tau) \left[\int_{\mathbf{z}(\tau)}^{\infty} P(\bar{z}, \tau, t)^{1/(1-\eta)} dF(\bar{z}; \mathbf{z}(\tau)) \right] d\tau. \tag{23}$$

Denote the total measure of nests at time t with $\bar{N}(t) = N(t)/[g_H(1 - \nu_m)]$. Substituting $p(\bar{z}, \tau, t)$ from (11) and $n(\bar{z}, \tau, t)$ from (21) into the price index in (10) and then into (23), we get:[14]

$$w(t) = \frac{1}{\sigma} Z(t) \bar{N}(t)^{\eta-1} \bar{n}(t)^{\sigma-1}. \tag{24}$$

The fraction $1/\sigma$ is the labor share in revenue, $Z(t)$ is the productivity of the representative nest, and the terms with $\bar{N}(t)$ and $\bar{n}(t)$ capture the welfare gains from a greater measure of nests and varieties. Since $\eta > \sigma > 1$, nests are less substitutable than varieties and their measure disproportionately contributes to welfare.

Substituting in (12) and using $\bar{C}(t) = \sigma w(t) L(t)$, the flow profit of all varieties is $\pi(\bar{z}, \tau, t) = \bar{\pi}(t)$ where

$$\bar{\pi}(t) = (\sigma - 1) \frac{w(t) L(t)}{\bar{N}(t) \bar{n}(t)}. \tag{25}$$

Total profits in the economy, $(\sigma - 1) w(t) L(t)$, is equally shared among varieties which in total measure $\bar{N}(t)\bar{n}(t)$.

Growth rates Given the growth rates of $\mathbf{z}$ and $\bar{N}$ in (17) and (19) and $g_{\bar{n}} = g_H - g_{\bar{N}}$, equations (24), (15) and (25) give us the growth rates of production wages, total income $\bar{C}$, and profits $\bar{\pi}$. The income of researchers w_H (below) grows at the same rate as profits. We summarize these growth rates:

$$\begin{aligned}
g_{\mathbf{z}} &= \nu_z g_H \\
g_{\bar{N}} &= (1 + \nu_m) g_H \\
g_{\bar{n}} &= -\nu_m g_H \\
g_w &= [(\eta - 1)(1 + \nu_m) - (\sigma - 1)\nu_m + \nu_z] \, g_H \\
g_{w_H} &= g_{\bar{\pi}} = g_L - g_H + g_w \\
g &= g_L + g_w
\end{aligned} \tag{26}$$

The earnings of production labor and researchers grow at the same rate, $g_{w_H} = g_w$, if their populations grow at the same rate, $g_H = g_L$.

[14]With these substitutions, the price index is

$$\begin{aligned}
1 &= \left[\sigma w(t) \bar{n}(t)^{1-\sigma} Z(t)^{(\sigma-1)/(\eta-\sigma)}\right]^{1/(1-\eta)} \int_{-\infty}^{t} N(\tau) \left[\int_{\mathbf{z}(\tau)}^{\infty} [\bar{z}\psi(t-\tau)]^{1/(\eta-\sigma)} \, dF(\bar{z}; \mathbf{z}(\tau))\right] d\tau \\
&= \left[\sigma w(t) \bar{n}(t)^{1-\sigma} Z(t)^{-1}\right]^{1/(1-\eta)} \frac{N(t)}{g_H(1+\nu_m)}
\end{aligned}$$

The integral in the first line is also in (20), from which we get the second line. Rearranging yields (24).

4.2 Allocation of research

Denote with $H_1(t)$ the total measure of variety researchers (the integral in (16)). The evolution of varieties in (4) applied to the aggregate economy implies:

$$[\bar{n}(t)\dot{\bar{N}}(t)] \;\; = -\delta\,[\bar{n}(t)\bar{N}(0)] + \frac{H_1(t)}{f_1}.$$

Since the growth rate of total varieties $\bar{n}(t)\bar{N}(t)$ is g_H, we get

$$H_1(t) = f_1(g_H + \delta)\bar{n}(t)\bar{N}(t). \tag{27}$$

Discounted as in (13), the value of a variety in any existing nest at time t is $v(\bar{z},\tau,t) = \bar{v}(t)$:

$$\bar{v}(t) = \frac{(1-\chi)}{\delta + \rho + g_H}\bar{\pi}(t). \tag{28}$$

where we use $r = \rho + g_{\bar{C}}$ and $g_{\bar{\pi}} = g_{\bar{C}} - g_H$. Substituting the evolution of varieties (21) into (14) and then into (5), the value of researching nests at time t is

$$\mathcal{V}(t) = \chi\frac{\Upsilon_1\Upsilon_3}{\Upsilon_2}m(t)\bar{\pi}(t)\bar{n}(t). \tag{29}$$

where

$$\Upsilon_3 = \int_0^\infty e^{-(\rho+g_H+\tilde{g})s}\psi(s)^{1/(\eta-\sigma)}ds$$

which exists by Assumptions A1(i) and A2. Equilibrium in the research market requires $\bar{v}(t)/f_1 = \mathcal{V}(t)$. Equating (28) to (29), and using (27) and $N(t) = m(t)H_2(t)$, we get the allocation of research:

$$\frac{H_1(t)}{H_2(t)} = \Upsilon_4\frac{1-\chi}{\chi}\left(\frac{\delta + g_H}{\rho + \delta + g_H}\right) \tag{30}$$

where

$$\Upsilon_4 = \frac{\int_0^\infty e^{-[\rho+(1+\nu_m+\nu_z/(\eta-\sigma))g_H]s}\psi(s)^{1/(\eta-\sigma)}ds}{\int_0^\infty e^{-(1+\nu_m+\nu_z/(\eta-\sigma))g_H s}\psi(s)^{1/(\eta-\sigma)}ds}$$

The number of researchers working on new varieties relative to nests is proportional to their profit shares $(1-\chi)/\chi$. It is decreasing in the discount rate ρ and increasing in the death rate of varieties δ, in the growth rate g_H as well as in, $1 + \nu_m + \nu_z/(\eta - \sigma)$, a term that captures the externalities in the research of new nests.

Equation (30) confirms that the share of research allocated to variety and nest creation is constant along the balanced growth path. Given the ratio $H_1(0)/H_2(0)$, market clearing for research labor, $H_1(0) + H_2(0) = H(0)$, determines the allocation of research at $t = 0$.

Productivity shifter $\mathbf{z}(0)$ and measures of new nests $m(0)$ and $N(0)$ are given by (17), (18) and (19) respectively. The wage of production labor $w(0)$ is in (24) with $Z(0)$ from (20), and total output $\bar{C}(0)$ is given by (15). The flow profit $\bar{\pi}(0)$ is in equation (25) and the wage of researchers $w_H(0) = \bar{v}(0) = \mathcal{V}(0)$ is in (28) and (29). Together with the growth rates in (26), this completes the characterization of the balanced growth path.

4.3 Social Planner

As with other semi-endogenous growth models, the planner can influence the level of income but not its growth. Substituting (17), (19), (20) and (27) into (24) and then into (15), we write total real consumption as a function of the allocation of research:[15]

$$\bar{C}(0) = \Upsilon_5 L(0) H_2(0)^{\nu_z + (1+\nu_m)(\eta-\sigma)} H_1(0)^{\sigma-1} \tag{31}$$

The solution to maximizing $\bar{C}(0)$ in (31) subject to $H_1(0) + H_2(0) = H(0)$ is:

$$\left(\frac{H_1}{H_2}\right)^{\text{planner}} = \frac{\sigma - 1}{\nu_z + (\eta - \sigma)(1 + \nu_m)}$$

The numerator captures the welfare gain from a wider range of varieties. In the denominator, ν_z is the productivity effect from increasing H_2, and $(1 + \nu_m)$ is the increase in the range of nests, whose welfare effect $(\eta - \sigma)$ is the direct effect $(\eta - 1)$ attenuated by the decrease in varieties per nest $(\sigma - 1)$.

If the planner can choose χ, through a policy on intellectual property, then he can attain first best in the market solution (30).[16]

5. ESTIMATION: LINKING THE MODEL TO PATENT DATA

Patent data are classified into 650 classes (four-digit CPC codes). To estimate the model, we use data on the number of patents by class for each year from 1875 to 2015. In the model, we assume that there is a continuum of patent classes with measure J. Each nest is specific to a class j, and patents are proportional to the number of new varieties. Nests arrive in each class with a Poisson rate $\lambda(t) = N(t)/J$ at time t.

Since the Poisson rate $\lambda(t)$ grows at a constant rate, each class at time t has only a finite number of nests (see Section 6.1.1 below). Let Ω_j be the set of nests in class j.

[15] The constant $\Upsilon_5 = (\eta-\sigma)^{-\kappa}[f_1(g_H+\delta)]^{1-\sigma}[g_H(1+\nu_m)/\Upsilon_2]^{\sigma-\eta}(g_H+\phi)^{-\nu_z-\nu_m(\eta-\sigma)}\tilde{f}_2^{\sigma-\eta-1}$.

[16] An alternative setting of χ is to allow the nest inventor to give a take it or leave-it offer to variety creators of the share χ of profits that she would retain. In this setting, the allocation of research is:

$$\left(\frac{H_1}{H_2}\right)^{\text{market}} = \Upsilon_4 \frac{\eta - 1}{\eta - \sigma}\left(\frac{\delta + g_H}{\rho + \delta + g_H}\right).$$

It is different from the planner, but the two cannot be ranked in general.

Combining equations (21), (22) and (27), the number of patents in patent class j at time t relative to the total number of patents is

$$s_j(t) = \frac{1}{H_1(t)} \sum_{\omega \in \Omega_j} h_1(\bar{z}(\omega), \tau(\omega), t) = \sum_{\omega \in \Omega_j} a(\omega) e^{-\beta_1 (t - \tau(\omega))} \tilde{\psi}(t - \tau(\omega)) \tag{32}$$

where $\beta_1 = g_H + \tilde{g}$,

$$a(\omega) = \frac{1}{\Upsilon_2 (g_H + \delta) \bar{N}(\tau)} \left(\frac{\bar{z}(\omega)}{\mathbf{z}(\tau(\omega))} \right)^{1/(\eta - \sigma)}, \tag{33}$$

$$\tilde{\psi}(t) = \left[\delta - \tilde{g} + \frac{\dot{\psi}(t)}{(\eta - \sigma)\psi(t)} \right] [\psi(t)]^{1/(\eta - \sigma)}. \tag{34}$$

In (32), the evolution of the number of patents over time follows the same shape for all nests, determined by β_1 and function $\tilde{\psi}$. The nest-specific parameters $\tau(\omega)$ and $a(\omega)$ determine, respectively, the birth of the nest and height of its peak. Note that these shares $s_j(t)$ are the model's prediction for each class j in the y-axis of Figure 1 above.

Assume that the shares in (32) are observed with measurement error, $s_j(t) + \epsilon_j(t)$, where $\epsilon_j(t)$ is independently and normally distributed with mean zero and variance ϑ_j^2.[17] Two econometric issues arise in estimating (32). First, the data are censored at zero. Whenever $\epsilon_j(t)$ is large and negative, we observe $s_j(t) = 0$. Second, we do not observe the number of elements in Ω_j. We address these issues by constructing a log-likelihood function similar to a Tobit model and using a likelihood-ratio test for the number of nests in each patent class.

Denote by $\widehat{s}_j(t; \beta, \Xi_j^I)$ the model's predicted shares in (32), where β are the parameters that are common to all nests, β_1 and those of function $\tilde{\psi}$, and $\Xi_j^I = \left\{ \vartheta_j, \{a(\omega), \tau(\omega)\}_{\omega \in \Omega_j} \right\}$ are the parameters that are specific to the class j, where superscript I indicates the number of nests in j, $I = |\Omega_j|$. The number of elements in Ξ_j^I, $2I + 1$. The probability of observing $s_j(t)$ is

$$\ell^I(s_j(t); \beta, \Xi_j^I) = \begin{cases} \frac{1}{\vartheta} \phi \left(-\frac{1}{\vartheta} \left[s_j(t) - \widehat{s}_j(t; \beta, \Xi_j^I) \right] \right) & \text{if } s_j(t) > 0 \\ \Phi \left(-\frac{1}{\vartheta} \widehat{s}_j(t; \beta, \Xi_j^I) \right) & \text{if } s_j(t) = 0 \end{cases}$$

where ϕ and Φ are respectively the density and the cumulative distribution function of

[17] There are many potential sources of measurement error. For example, the patent office may spuriously grant patents that have no market value, and some varieties may need more or fewer patents.

a standard normal. The log-likelihood function of the vector s_j for $t = 1, ..., T$ is

$$\mathcal{L}_j^I(s_j; \beta, \Xi_j) = \frac{1}{T} \sum_{t=1}^{T} \log \ell^I(s_j(t); \beta, \Xi_j^I).$$

The likelihood ratio:

$$-2 \left[\mathcal{L}_j^{I+1}(s_j; \beta, \Xi_j^{I+1}) - \mathcal{L}_j^I(s_j; \beta, \Xi_j^I) \right] \tag{35}$$

is distributed chi-squared with two degrees of freedom under the hypothesis that $|\Omega_j| = I$.

5.1 Patents: Estimation procedure

Stage 1: β and Ξ_j^I. To identify parameters β governing the evolution of patents over time within nests, we select a subsample of CPC codes where most of this life cycle occurs within the sample. We calculate the average $s_j(t)$ in the first and last ten years of our data (1875–1884 and 2006–2015), and select the 250 CPC codes (out of 650) whose maximum share $s_j(t)$ is three times higher than these averages. For the example of computers in Figure 1(A) above, only patent class G06C is included in the estimation of β, and for organic chemistry in Figure 1(B), only patent class C07K is excluded.

We iterate over steps (i) and (ii) until the number of nests per CPC and β are stable.[18]

(i) Fix a guess of β. Separately for each CPC code j and starting with $I = 0$, we choose Ξ_j^I to maximize $\mathcal{L}_j^I(s_j; \beta, \Xi_j^I)$ and Ξ_j^{I+1} to maximize $\mathcal{L}_j^{I+1}(s_j; \beta, \Xi_j^{I+1})$. We say that a CPC code j has I nests when we cannot reject the null hypothesis $|\Omega_j| = I$ at a 95% confidence level using the likelihood ratio test in (35). If we reject $|\Omega_j| = I$, we repeat the exercise for $I + 1$ until we cannot reject the null.

(ii) Fix the number of nests I_j in each CPC code j. We choose β to solve:

$$\max_{\beta} \frac{1}{J} \sum_{j=1}^{J} \max_{\Xi^{I_j}} \mathcal{L}_j^{I_j}(s_j; \beta, \Xi^{I_j}).$$

That is, for each guess of β, we iterate over guesses of Ξ^{I_j} for each j.

Once the algorithm converges, we obtain our estimate of β. Then, for each of the 650 CPC codes (not only those used to estimate β), we repeat step (i) above. This gives us a collection of nest shifters $a(\omega)$ and birth dates $\tau(\omega)$ for each patent class.

[18]The log likelihood is always increasing in the number of nests. Separating steps 1 and 2 ensures that the algorithm does not select a β with a worse fit in order to increase the number of nests.

Stage 2: Arrival of nests The arrival rate of nests in a patent class in year t is Poisson with parameter $\lambda(t) = \lambda(0)e^{g_H(1+\nu_m)t}$. For each year t, let $k(t)$ be the share of patent classes with at least one birth date $\tau(\omega)$ in $[t, t+1)$. We choose the parameters $\lambda(0)$ and $g_H(1+\nu_m)$ to maximize:

$$\frac{1}{T}\sum_t (1-k(t))e^{-\lambda(t)} + k(t)[1-e^{-\lambda(t)}]$$

where $e^{-\lambda(t)}$ is the probability of not observing a birth in year t.

Stage 3: Nest heterogeneity. We use the estimate of $g_H(1+\nu_m)$ from stage 2 and the birth $\tau(\omega)$ and height $a(\omega)$ of each nest in stage 1 to calculate $\tilde{a}(\omega) = a(\omega)e^{\tau(\omega)(1+\nu_m)g_H}$. By equation (33), $\tilde{a}(\omega)$ is distributed log-normally with standard deviation parameter $\theta/(\eta-\sigma)$. Our estimate of $\theta/(\eta-\sigma)$ is the standard deviation of $\log\tilde{a}(\omega)$.

Parametrization. We set $g_L = g_H = 0.011$ to the growth rate of patents from 1875 to 2015, and the growth rate of income per capita to $g_w = 0.02$.[19] We cannot identify the demand parameters from our data. We take the elasticity of substitution between varieties within nests, $\sigma/(\sigma-1) = 8$, from quantitative applications of nested CES models, and the elasticity between nests, $\eta/(\eta-1) = 2.5$, from Grant and Soderbery (2024).[20] Together, these parameters imply an obsolescence rate in (32) of $\beta_1 = 0.035$. We set the death rate of varieties to $\delta = 0.10$.[21] We parameterize

$$\psi(t) = 1 - \exp\left(-\beta_2 t^{\beta_3}\right) \tag{36}$$

the cumulative distribution of a Weibull distribution.[22] With this parametrization, there are two economy-wide parameters $\beta = \{\beta_2, \beta_3\}$ governing the evolution of patents

[19] We regress $\log(\text{patents}_t) = \alpha + g_H t + \epsilon_t$ and obtain $\widehat{g}_H = 0.011$ (s.e. = 0.0006, $R^2 = 0.72$). This growth rate is approximately the growth rate of the US population, as it is well known.

[20] See Atkeson and Burstein (2008), Eaton, Kortum, and Sotelo (2013), Edmond, Midrigan, and Xu (2015), and Gaubert and Itskhoki (2021) for the elasticity between varieties. They use a smaller elasticity between nests because they interpret nests as industries, whereas we interpret them as types of goods within industries. Using a level of aggregation similar to ours (SITC four digit), Grant and Soderbery (2024) estimate an elasticity of US import demand between 2.2 and 2.7. This elasticity should be similar to ours if goods from different countries are of different nests. Our elasticities imply $\sigma = 1.11$ and $\eta = 1.667$, which are respectively close to the median markup (1.14) and 99th percentile markup (1.76) in Edmond, Midrigan, and Xu (2015).

[21] Below, we interpret establishments in the Census data as collections of varieties in the model. The death rate of varieties should be close to the death rate of small firms in Haltiwanger, Jarmin, and Miranda (2013) (between 0.09 and 0.20) and higher than the death rate of firms in Peters and Walsh (2021) (0.07 in recent years).

[22] In Appendix B.1, we set $\psi(t) = t^\gamma$ so that the evolution of varieties in (4) has the shape of the probability density function of a gamma distribution. With an unbounded ψ, matching the fast decline in the number of patents after a boom requires a high estimate for β_1 associated with an implausibly high growth rate $g_w = 0.20$! For this reason here we parameterize ψ with a bounded function and impose the growth rate of the economy g_w. The two parameterizations have a similar fit, and almost identical estimates for stages 2 and 3, showing the robustness of the results to different functional forms.

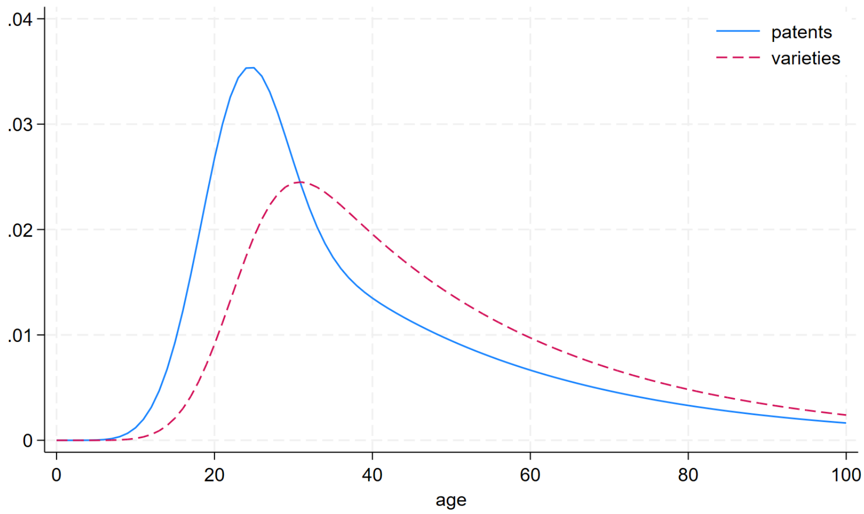


FIGURE 5: Evolution of the number of patents and varieties after the birth of a nest

Notes: The figure plots the evolution of the number of patents and varieties in a nest after birth implied by the estimates of β_2 and β_3. This evolution is the same for all nests, whereas the height of each curve depends on the nest-specific parameter $a(\omega)$. Here, we normalize the two curves to have the same integral.

over time within nests, function $\tilde{\psi}$ in (34).

5.2 Patents: Estimation results

In stage 1, the estimated $\beta_2 = 1.15e - 05$ (s.e. 1.5e-7) and $\beta_3 = 3.64$ (s.e. 0.005). Figure 5 illustrates the implied evolution in the number patents and varieties after the birth of a nest. The number of patents peaks 25 years after birth, whereas the number of varieties peaks at 31 years and takes longer to decline. All nests follow this same evolution.

Stage 1 also yields the number of nests per patent class and the class-specific parameters Ξ_j^I. Table 2 reports the distribution of number of nests per class. The average is 4.34 and the median is four.[23] Although we target the likelihood rather than the R-squared, Table 3 reports the distribution of R-squared across patent classes to assess the model's fit. For each class, there are 141 observations (one per year) and on average 8.7 parameters (two parameters, $\tau(\omega)$ and $a(\omega)$, per nest). The tenth percentile R-squared is 0.49 and the median is 0.80. Figure 1 illustrates with a line the predicted path of patents for the model in the examples of CPC codes in G06, computing, and in C07, organic chemistry.

Figure 6 summarizes the nest-specific parameters $\tau(\omega)$ and $a(\omega)$. In panel (A), we plot $k(t)$, the share of patent classes per year with the birth of a nest. Since our data go from 1875 to 2015, the figure plots nests born between 1850 and 1990, whose peak is in the period of our data. Nests are rare events. On average, only 2.8 percent of classes receive

[23]For computational reasons, we estimate the model with at most $I = 6$ nests per CPC code in estimating β, and at most $I = 10$ nests in estimating Ξ_j^I. The highest statistical test is $I = 9$.

TABLE 2: Distribution of number of nests per patent class

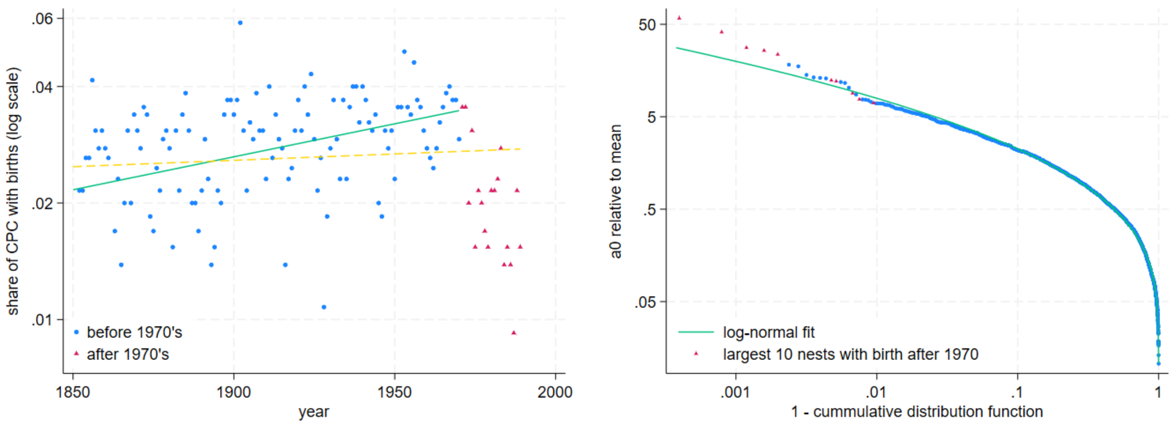
number of nests (k)	number of patent classes with k nests	cumulative perc.
0	1	0.15
1	29	4.6
2	60	14
3	98	29
4	165	54
5	143	76
6	93	91
7	44	97
8	14	99.5
9	2	99.8
≥ 10	1	100
total	**650**	

Note: The table reports the number of patent classes (four-digit CPC codes) with 0, 1, 2, ... nests. The average number of nests per patent class is 4.34.

TABLE 3: Percentiles and mean of the distribution of R-squared across CPC codes

percentiles					
10%	25%	50%	75%	90%	**mean**
0.49	0.65	0.80	0.90	0.95	0.75

Stage 1 estimates the model separately by CPC code given the common parameters β. The table presents moments from the distribution of R-squared (not targeted) across CPC codes. For each CPC code, there are 141 observations (number of years) and the number of parameters on average is 8.7 (two times the number of nests, excluding the standard error ϑ, which does not enter the fit).



(A) Share of CPC codes with the birth of a nest (B) Distribution of $\tilde{a}_0(\omega)$

FIGURE 6: An illustration of the nest-specific parameters $\tau(\omega)$ and $a(\omega)$ from step 2

a nest in a given year. There is a clear, albeit small, increase in the number of births per year up until 1970 (observations in blue circles). Applying Stage 2 to this period, 1850 to 1970, we estimate a Poisson parameter $\lambda(1970) = 0.035$ (s.e. 0.002) and a growth rate of births $g_N = g_H(1 + \nu_m) = 0.0034$ (s.e. 0.0004). Panel (B) plots the distribution of $\tilde{a}(\omega)$, which is proportional to productivity $[\bar{z}(\omega)/\mathbf{z}(\tau(\omega))]^{1/(\eta-\sigma)}$, together with the log-normal fit. The standard deviation of $\log \tilde{a}(\omega)$ is $\theta/(\eta - \sigma) = 1.21$. It implies a 90-10 ratio of 22 or 2.6 log points.

In Figure 6(A), the number of estimated nest births declines after 1970 (observations in red triangles). We conjecture that there is no actual decline in births but that nests are now more frequent in a few patent classes, such as G06F and G06Q in Figure 1. This conjecture is consistent with panel (B) where all the nests with highest estimated $\tilde{a}(\omega)$ have births after 1970.[24]

Table 4 reports the implication of the parameters above for economic growth. By design, the growth rate of income per worker $g_w = 0.02$. Of this growth, 0.0023 (11 percent) is accounted for by the growth of number of new nests, 0.0011 (6 percent) by the growth of varieties per nest, and 0.017 (83 percent) by productivity growth. So although the growth rate of varieties is twice that of nests, their contribution to welfare is smaller because varieties are highly substitutable.

Economically, nests represent breakthrough ideas that give rise to a boom in innovation. Growth in productivity $\mathbf{z}$ captures the improvements in new nests with respect to old ones. While we model these improvements as productivity increases, they are isomorphic to quality improvements or to an expanded scope of production under a richer CES

[24] Appendix B.2 shows that the concentration of patents across classes sharply increases in the last years of our data. The estimation procedure does not identify nests born at similar times within patent classes. It identifies them as one nest with a high $a(\omega)$. Our estimated parameters imply that the probability of two nests being born in the same patent class and year is only 0.0002, but this distribution may have changed as some patent classes became "large" after 1970.

TABLE 4: Parameter estimates and decomposition of growth in real income per capita

	model expression	value
A. Parameters		
growth in total varieties	g_H	0.011
obsolescence rate of shares s_j (β_1)	$g_H(1+\nu_m+\nu_z/(\eta-\sigma))$	0.035
growth in nests	$g_H(1+\nu_m)$	0.0034
growth in varieties per nest	$-g_H\nu_m$	0.0077
elasticity of subst. between varieties	$\sigma/(\sigma-1)$	8
elasticity of subst. between nests	$\eta/(\eta-1)$	2.5
B. Decomposition of growth in w		
Welfare gains from...		
... growth in nests	$(\eta-1)(1+\nu_m)g_H$	0.0023
... growth in varieties per nest	$-(\sigma-1)\nu_m g_H$	0.0011
... growth in nest productivity	$\nu_z g_H$	0.017
total	g_w	**0.020**

preference structure.[25] The decomposition in Table 4 implies that newer breakthroughs render the old ones obsolete not because they have become more common (11 percent of growth), nor because each boom now contains more patents (6 percent of growth), but mostly because the newer breakthroughs are better—in productivity, quality, or scope—than the ones they replace.

6. OUT-OF-SAMPLE ANALYSIS

In Section 6.1, we compare the predictions of the model to the moments from the data highlighted in Section 2 above, as well as similar moments from the Census of Manufactures and CBP data sets. In Section 6.2, we scrape data on the text of 2.8 million patents to construct the measure of patent influence proposed by Kelly, Papanikolaou, Seru, and Taddy (2021). We then compare the timing of the birth of productive nests in the estimation to the filing of the most influential patents of a class.

6.1 Non-targeted moments on patents, CBP and Census

Table 3 summarizes the fit of the model given the nest-specific parameters $a(\omega)$ and $\tau(\omega)$. In this section, we ask how well the model fits the empirical regularities of Section 2 given the estimated distributions of $a(\omega)$ and $\tau(\omega)$. We simulate 650 patent classes (the number of CPC classes in the data) using the five estimated parameters $\lambda(1970)$, g_N, θ, β_2 and β_3, and the four calibrated parameters δ, β_1, η and σ.

[25] That is, adding a sub-nest to each variety.

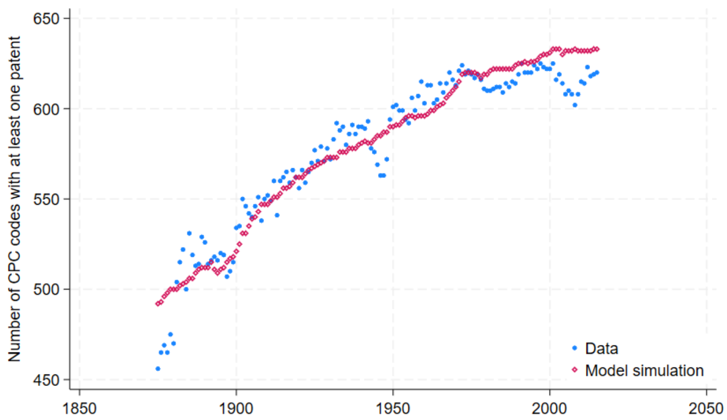


FIGURE 7: Number of active CPC codes in the data and model simulations

6.1.1 Model simulation procedure

Birth of nests Denote the last year of our data $T = 2015$ and take $\lambda(t) = \lambda(T)e^{g_N(t-T)}$ for all t. Consider the time of birth of the nests in a class $\tau_1 > \tau_2 > ...$ with $\tau_i \leq T$. Starting with $\tau_0 = T$ and conditional on τ_{i-1}, the probability that the time interval $(\tau_{i-1} - \tau_i)$ is greater than or equal to Δ is

$$\text{Prob}\left(\tau_{i-1} - \tau_i \geq \Delta | \tau_{i-1}\right) = \exp\left(-\int_{\tau_{i-1}-\Delta}^{\tau_{i-1}} \lambda(\tau)d\tau\right) \quad = \exp\left[-\frac{\lambda(\tau_{i-1})}{g_N}\left(1 - e^{-g_N\Delta}\right)\right]$$

For each patent class, we generate a sequence of nest birth times $\tau_1, \tau_2, ...$ from this distribution. Note that the number of births is always finite, since the probability that $\tau_{i-1} - \tau_i = \infty$ is positive, decreasing in τ_{i-1} and tends to one as $\tau_{i-1} \to -\infty$.[26]

Productivity of nest For each nest birth τ_i above, we draw the shifter $\tilde{a}_i$ from a log normal with mean parameter 0 and standard deviation parameter $\theta/(\eta - \sigma)$ and take $a_i = \tilde{a}_i e^{-\tau_i g_N}$. A nest is a birth-productivity shifter pair (τ_i, a_i) in a CPC class.

Patent shares With pairs of birth and shifters (τ_i, a_i) we can construct the number of patents per class using (32) given ψ in (36), and calculate the share of patents in each of the 650 simulated classes and years 1875, ..., 2015.

6.1.2 Simulation and patent data

Although we do not directly target the moments in Figures 2, 3, and 4 of Section 2, they are related to the boom-and-bust cycles within the CPC codes targeted in our estimation. Figure 7 compares the predicted number of CPC codes with at least one

[26] Although we could make draws until $\tau_{i-1} - \tau_i = \infty$, we stop the algorithm at $\tau_{i-1} \leq 1775$, 100 years before the period 1875-2015 used to construct moments.

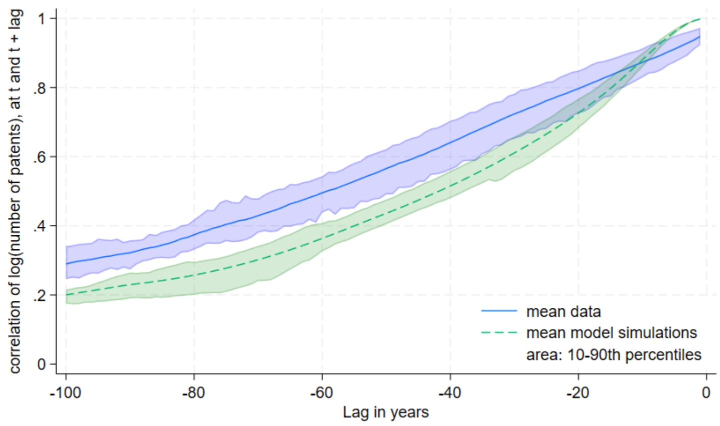
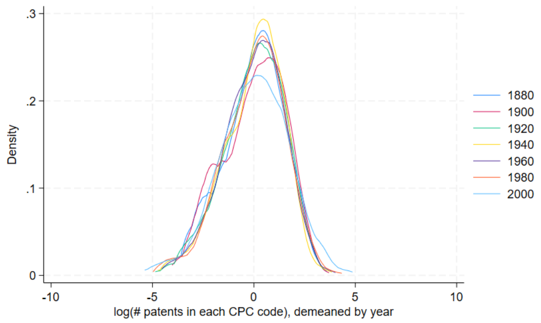
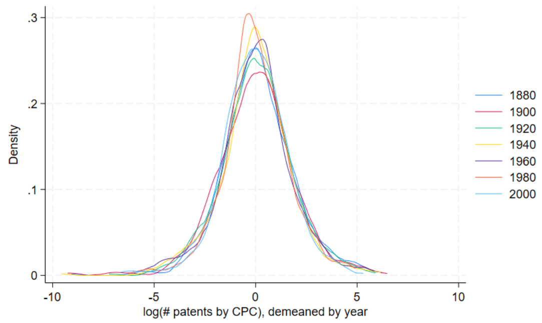


FIGURE 8: Correlation of the number of patents (in logs) and its lagged variable



(A) Data



(B) Model simulations

FIGURE 9: Kernel densities of the distribution of number of patents (in logs and demeaned) across CPC codes in the data and model

patent per year in the model simulations to the data (repeated from Figure 2).[27]

To capture the patterns in Figure 3, for each year t and time lag $\ell = 1, ..., 100$, we calculate the correlation between the (log) number of patents in t and in $t - \ell$ across patent classes. Figure 8 plots the mean, 10^{th} and 90^{th} percentiles of these correlations in the data and model. The correlation declines faster in the model than in the data, suggesting that the model generates more churning across patent classes than the data.

Figure 9(A) shows the density of number of patents (in logs and demeaned) across patent classes in the data for every two decades, repeated from Figure 4. Relative to the data, the distribution from the model in Figure 9(B) has a thicker right tail, but the two figures display similar stability across decades and almost the same standard deviations (1.44 in the model and 1.43 in the data).

[27]Both the data and the model have classes with fractions of a patent. In the data, when patents are classified into more than one CPC code, say 3, we count it as 1/3 of a patent in each of the codes. In the model, the measure of patents is continuous and minuscule for nests that are 100 years old or more.

6.1.3 Simulation, CBP, and Census of Manufactures

The CBP has yearly data on employment and establishments from 1959 to 2016, and the Census of Manufactures has data on value added and establishments for every ten years from 1870 to 1940.[28] These data are not amenable to the identification of nest births and heights in stage 1 of the estimation of Section 5 or to the birth of new industries in Figure 7 because the data time span is shorter, industry classifications are broader, and the data exhibit discontinuities at the industry level in the years where the industry classifications change.[29] Still, these data provide useful information on the shifts of real economic activity across industries and time.

Value added and production labor in a nest are proportional to the number of varieties. So, to construct the model's predictions for these variables, we use the estimated path of varieties in a nest in Figure 5 and randomly group the simulated nests above into 289 industries, where 289 is the number of industries in the CBP data.[30]

Figure 10(A) compares the lagged auto-correlation in employment in the model to the CBP data on employment and to the Census data on value added. The correlations in the model lie in between those of the CBP and the Census. Similar to Figure 8 for patents, the autocorrelation decreases as the time lag increases.

Appendix Figure 10(B) plots the density of employment (in logs and demeaned) for each decade in the CBP data, and the pooled sample for the model. These densities are less stable in employment than in patent data (Figure 9), and the spread is larger in the CBP data than in the model. For example, the standard deviation in log employment in a typical year is 1.86 in the data and 1.15 in the model.

We also decompose value added and employment into the number of establishments

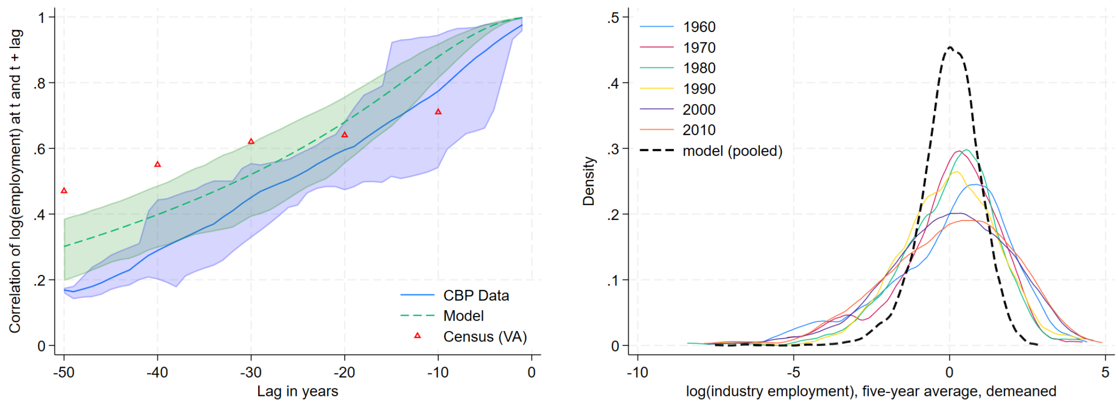
[28] Here, we do not use the Census from 1860 because we use patent data from 1875 onward. We do not use CBP data between 1947 and 1959 because the number of industries is smaller and varies a lot in these years, sometimes as low as 26 (1956).

[29] When new patent classes are created, Google Patents retroactively reclassify patents into the new categories so that the birth of patent classes is smooth, as illustrated in Figure 1. With CBP it is not feasible to reclassify establishments that existed many decades past. As a second best, we use a weighting scheme following Eckert, Fort, Schott, and Yang (2021b) to harmonize industries but still obtain significant discontinuities whenever industry classifications change as 1974 and 1998.

[30] A nest ω in the simulation is characterized by its productivity parameter $a(\omega)$ and its birth $\tau(\omega)$. Using equation (21) and the definition of $a(\omega)$ in (33), the share of varieties in a nest relative to total varieties is

$$\frac{n(\omega,t)}{\bar{N}(t)\bar{n}(t)} = (g_H + \delta)a(\omega)\psi(t-\tau(\omega))^{1/(\eta-\sigma)}e^{-\beta_1(t-\tau(\omega))}.$$

We calculate these shares using the estimates of β_1 and of β_2 and β_3 defining function ψ in (36).



(A) Data

(B) Model simulations

FIGURE 10: Kernel densities of the distribution of number of patents (in logs and demeaned) across CPC codes in the data and model

Note: In panel A, for the model and the CBP data, we calculate the log of employment in each year and industry. For each year t and lag ℓ, we calculate the autocorrelation between employment across industries. The figure plots the mean and the 10-90 percentiles (shaded area) of these correlations as a function of the lag ℓ (x-axis). We repeat the exercise for value added in the Census of Manufactures. In Panel B, we calculate the log of employment and demean it in each year for the CBP data. The figure displays the kernel density of the average of five years around the turn of the decade (e.g., 1968-1972). We follow the same procedure for the model, but only present the density pooled across all decades.

and the size of establishments as follows:

$$\text{\# workers} = \text{\# establishments} \times \frac{\text{\# workers}}{\text{\# establishments}}$$
$$\text{value added} = \text{\# establishments} \times \frac{\text{value added}}{\text{\# establishments}}$$

Tables A7 and A8 present the results for the first decomposition using CBP data and the second using Census data. All regressions have industry and year fixed effects. The results are remarkably similar. Growth in the number of establishments accounts for 0.55 of the growth in employment and for 0.60 of the growth in value added, with the remaining growth accounted for by the size of establishments. In a cross section, the number of establishments accounts for 0.81 of the variation in employment across industries, and for 0.79 of the variation in value added.

Following Klette and Kortum (2004), we assume that establishments in the model are random collections of varieties, and that they only produce varieties for a given nest. Assume the number of varieties per establishment is distributed Poisson with parameter n^{α} when the number of varieties in the nest is n. With $\alpha = 0.77$, the model exactly matches the share of value added growth accounted for by growth in the number of establishments, 0.60. The model then predicts that the number of establishments in a cross section accounts for 0.66 of the variation in value added and of employment in the data. So, in both the data and the model, the number of establishments accounts for a larger share of the variation in industry size (value added or employment) than its growth, but the share is larger for the data than the model.[31]

In all, the model matches well out-of-sample moments related to the shift of economic activity across industries and the decomposition of economic growth at the industry level into growth in number of establishments and the size of establishments.

6.2 Textual analysis of patents

The model estimation identifies the timing and productivity of breakthroughs from patent *counts* alone. Here we bring independent evidence from the *text* of patents. In a recent influential paper, Kelly, Papanikolaou, Seru, and Taddy (2021) (KPST, henceforth) postulate that the text in the most influential patents is uncorrelated with past patents and correlated with future patents. In particular, they propose the following measure of the influence of patent j:

$$\text{KPST}_j = \frac{\sum_{i \in \mathcal{F}_j} \rho_{ij}}{\sum_{i \in \mathcal{B}_j} \rho_{ij}} \tag{37}$$

where ρ is a correlation of a vector of patent words, adjusted to downplay common English terms, $\mathcal{B}_j$ is the set of patents preceding j and $\mathcal{F}_j$ is the set following j.

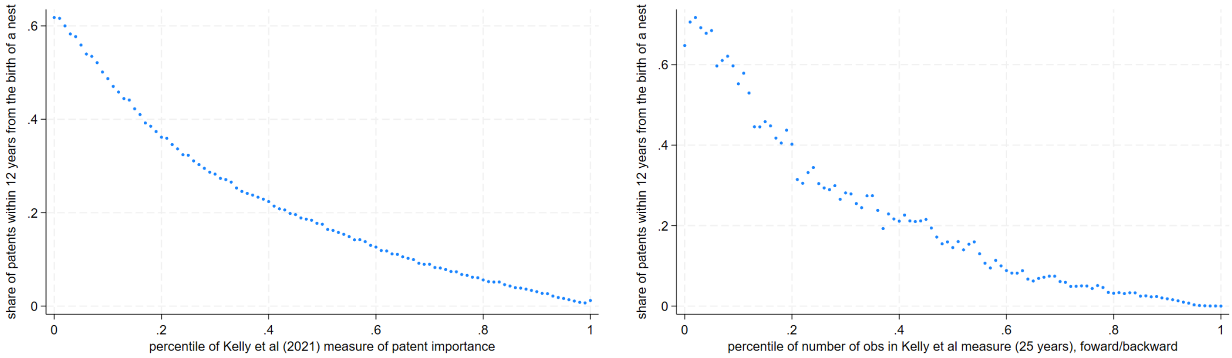
We scrape data on more than 2.8 million patents and apply this measure separately to each patent class.[32] There is an average of about 3,500 patents per patent class. We rank patents within classes from the most to the least influential and group them into percentile bins (with zero being the most influential). We then calculate the share of patents in each bin that are within the first 12 years after the birth of a nest in the estimated model, where twelve years is the period where the first percentile of varieties in a nest are created (Figure 5). Figure 11(A) illustrates a tight relation between these two variables, KPST's measure and our estimated nest births.

The regressions in Appendix Table A5 confirm that KPST is associated not only with nest birth but also with nest productivity $\tilde{a}(\omega)$, after controlling for year and CPC code fixed effects. For robustness, we also use patent citations as well as variations in the KPST measures (changes in the time span of $\mathcal{F}_j$ or three-digit CPC codes).

These measures of influence, however, capture moments not used in the estimation,

[31] See Appendix D for details, and for the further decomposition of value added per establishment into number of workers per establishments and value added per worker.

[32] We take the backward period to be 5 years as in KPST and the forward period to be 25 years, where 25 years corresponds to the period between the birth of a nest and peak innovation in our estimated model (Figure 5). KPST apply their measure to all CPC codes for a shorter forward period, only 10 years, due to computational constraints. Their objective was to detect the most influential patents of all times, whereas ours is to identify breakthroughs within narrow patent classes. Our results are unchanged if comparison set is a three-digit CPC code instead of four-digit code (see Appendix Figure A5 and column (7) of Table A5).



(A) KPST measure

(B) Number of observations $|\mathcal{F}_j|/|\mathcal{B}_j|$

FIGURE 11: Most influential patents and birth of nests

Note: The x-axis in (a) is the ranking of KPST measure within four-digit CPC code grouped into percentile bins. In (b), the x-axis is the ranking (also in bins) of the ratio of the number of observations used to construct the KPST measure, $\mathcal{F}_j/\mathcal{B}_j$. There's an average of about 3500 patents per CPC code. On the y-axes is the share of patents in each percentile bin that is within the first 12 years of the birth of a nest or in the year of the birth minus one. Of all patents, 0.21 are within this interval of a birth.

such as patent texts and citations, as well as the patent count used in the estimation. For example, 70 percent of the variation in KPST in a typical CPC code is due to the ratio $|\mathcal{F}_j|/|\mathcal{B}_j|$, and 30 percent due to the average correlation.[33] In Figure 11(B), we replace KPST with the ratio $|\mathcal{F}_j|/|\mathcal{B}_j|$ on the x-axis and obtain a similar graph.

In sum, the births of productive nests in the estimated model are tightly associated to the dates of the most influential patents, though there is overlap in the information used in our estimates and in constructing these measures of patent influence.

7. AGGREGATE FLUCTUATIONS

Breakthroughs are rare and highly heterogeneous in productivity in the estimated model. We simulate the model out of the balance growth path to illustrate its implications for aggregate economic fluctuations.

In Section 7.1, we cast the model into changes to avoid the need to estimate variables that are difficult to measure, such as f_1, f_2, H, L, $\mathbf{z}$, and $\bar{n}$. In Section 7.2, we study impulse responses to a technology shock. In Section 7.3, we conduct multiple simulations of the economy with random draws of nests and their productivity.

[33]We write

$$\log \mathrm{KPST}_j = \log |\mathcal{F}_j|/|\mathcal{B}_j| + \log \frac{\sum_{i \in \mathcal{F}_j} \rho_{ij}/|\mathcal{F}_j|}{\sum_{i \in \mathcal{B}_j} \rho_{ij}/|\mathcal{B}_j|}$$

where $|\mathcal{X}|$ represents the number of elements in set $\mathcal{X}$, and the last term is the mean correlation of texts of patent j with the patents after it, divided by the mean correlation with the patents that preceded j. Separately for each CPC code, we perform a variance decomposition. The share of the term $\log |\mathcal{F}_j|/|\mathcal{B}_j|$ in the variance of $\log \mathrm{KPST}_j$ is 70 percent on average and 69 percent median.

7.1 Deviations from balanced growth

We recast the model in deviations from the balanced growth path, writing $\widehat{x}(t) = x(t)/x^*(t)$ for the ratio of each variable to its balanced-growth value. We hold the populations of workers and researchers on their balanced-growth paths, $\widehat{L}(t) = \widehat{H}(t) = 1$, to focus on the split between research of varieties and nests. We iterate over guesses of this split, $\widehat{H}_1(t)$, and check the research equilibrium below. The detailed derivations and the simulation procedure are in Appendix C.

Research allocation Given $\widehat{H}_1(t)$, $\widehat{H}_2(t)$ and $\widehat{\bar{H}}_2(t)$ are:

$$\frac{H_1^*}{H^*}\widehat{H}_1(t) + \left[1 - \frac{H_1^*}{H^*}\right]\widehat{H}_2(t) = 1. \tag{38}$$

$$\widehat{\bar{H}}_2(t) = (\phi + g_H)\int_{-\infty}^{t} e^{-(\phi+g_H)(t-s)}\widehat{H}_2(s)ds. \tag{39}$$

New nests The instantaneous Poisson arrival rate of nests at time t is

$$K(t) = \widehat{H}_2(t)\widehat{\bar{H}}_2(t)^{\nu_m g_H} N^*(t) \tag{40}$$

where $N^*(t) = J\lambda^*(t)$ is known from the estimation. Nest-specific productivity deviations, $\widehat{\bar{z}}(\omega)$ are distributed log-normal with mean parameter $\log\widehat{\mathbf{z}}(t)$, where

$$\widehat{\mathbf{z}}(t) = \widehat{\bar{H}}_2(t)^{\nu_z} \tag{41}$$

With these nest draws, we construct the following summary statistic:

$$\bar{N}(t)\widehat{Z(t)^{1/(\eta-\sigma)}} = \left(\frac{e^{-\nu_z g_H t}}{Y_2}\right)^{1/(\eta-\sigma)} \frac{1}{\bar{N}^*(t)} \sum_{\omega\in\Omega(t)} \left[\psi(t-\tau(\omega))\widehat{\bar{z}}(\omega)e^{\nu_z g_H \tau(\omega)}\right]^{1/(\eta-\sigma)}. \tag{42}$$

New varieties Given a path of $\widehat{H_1}(t)$ and an initial year t_0 with $\bar{N}\widehat{(t_0)\bar{n}}(t_0) = 1$, we construct the path of $\widehat{\bar{N}(t)\bar{n}(t)}$ by integrating:

$$\dot{\widehat{\bar{N}(t)\bar{n}(t)}} = (\delta + g_H)\left[\widehat{H_1}(t) - \widehat{\bar{N}(t)\bar{n}(t)}\right]. \tag{43}$$

Research The deviation in the value of research in varieties and nests is:[34]

$$(44)\qquad \widehat{v}(t) =(\delta+\rho+g_H)\widehat{w}(t)\mathbb{E}\int_t^{\infty} e^{-(\delta+g_H+\rho)(s-t)}\left[\widehat{\bar{N}(s)\bar{n}(s)}\right]^{-1} ds$$

$$(45)\qquad \widehat{\mathcal{V}}(t) =\Upsilon_6\widehat{\bar{H}}_2(t)^{\nu_m+\nu_z/(\eta-\sigma)}\widehat{w}(t)\times$$

$$\mathbb{E}\int_t^{\infty} e^{-[\rho+g_H+g_H\nu_z/(\eta-\sigma)](s-t)}\psi(s-t)\left[\bar{N}(s)\widehat{Z(s)^{1/(\eta-\sigma)}}\right]^{-1} ds.$$

Research is in equilibrium if $\widehat{v}(t)=\widehat{\mathcal{V}}(t)$. Because profits are equalized across nests, deviations in the value of varieties in (44) is the discounted deviation in the total number of varieties, which in turn depends on $\widehat{H}_1$ non-stochastically.[35] The value of nests in (45) has two terms. The term $\widehat{\bar{H}}_2(t)$ is the spillover of the research allocation into the productivity of nest researchers. The expectation term is a discounted measure of competition across nests, which depends on chance and on the allocation of research through its effect on the distributions of nests' arrival (in (40)) and productivity (mean $\widehat{\mathbf{z}}(t)$).

In addition to the parameter estimates, we need values for ρ, ϕ and H_1^*/H^*. We take the discount rate to be $\rho=0.05$, and experiment with different values for ϕ and H_1^*/H^*.

7.2 Impulse response functions

To understand the mechanism, we trace the response to a large technology shock illustrade in Figure 12(A). At $t=1970$, we set the technology draw for all nests to the 95^{th} percentile of the log-normal distribution with mean parameter $\log\mathbf{z}^*(t)$ and variance θ^2. For all other periods, we set the number of nests to the Poisson parameter $K(t)$ and their productivity term in (42) to $\mathbb{E}(\bar{z}^{1/(\eta-\sigma)}|\mathbf{z}(t))$.[36] Both K and $\mathbf{z}$ may change with the reallocation of research. The dashed line in Figure 12(A) shows the direct effect of the shock on the representative productivity $\widehat{Z}$ before the reallocation of research labor. From equation (24), real income $C(t)=w(t)L(t)$ in changes is

$$(46)\qquad \widehat{C}(t)=\widehat{Z}(t)\widehat{\bar{N}}(t)^{\eta-1}\widehat{\bar{n}}(t)^{\sigma-1}.$$

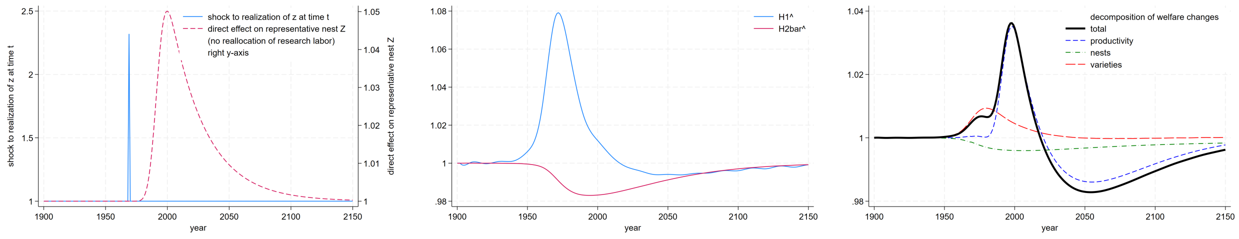
Without reallocation of research $\widehat{\bar{N}}=\widehat{\bar{n}}=1$, and the welfare changes equal $\widehat{Z}-1$. It peaks at 0.05 around the year 2000.

For didactic purposes, we focus on the case $\phi=0$ in (39) and $H_1^*/H=0.5$, so deviations in research are equal in magnitude for varieties and nests ($\widehat{H}_1-1=-(\widehat{H}_2-1)$ in (38)).

[34] $\Upsilon_6^{-1}=\int_0^{\infty}e^{-[\rho+g_H+g_H\nu_z/(\eta-\sigma)]t}\psi(t)dt$, so that $\widehat{V}(1,\tau)=1$ when $\widehat{\bar{N}}(t)\widehat{Z}(t)^{1/(\eta-\sigma)}=1$ for all t.

[35] The equations above are derived under the assumption that aggregate fluctuations are small enough to maintain entry across all existing nests in all periods.

[36] This approach implies that the term $N\widehat{Z^{1/(\eta-\sigma)}}$ in the value of nests (C.12) is at its BGP level when $\widehat{H}_2=1$ and $t\neq 1970$.



(A) Shock (B) Research reallocation (C) Welfare

FIGURE 12: Impulse response functions to a technology shock, $\phi = 0$ and $H_1^*/H^* = 0.5$

The anticipated rise in $Z(t)$ lowers the value of nests in (45), so researchers shift from nests toward varieties (panel B of Figure 12). The variety boom raises welfare modestly and early; the offsetting decline in nests is larger and more persistent—nests do not die and are less substitutable. So, the income gain peaks at 0.036, below the 0.05 direct effect (panel C). The reallocation of research leads to a prolonged decrease in $\bar{H}_2$ (panel B), which decreases the productivity of new nests in (41). The result is an endogenous downturn after the boom.

Appendix Figure A7 presents the impulse responses for other cases of parameters. When we shut down knowledge spillovers and set $\widehat{\bar{H}}_2 = 1$, the model generates an initial boom similar to the case above but no endogenous downturn (panels B and C).[37] The reallocation of research labor is stronger when nest researchers are a smaller share of the total: as H_1^*/H rises, $\widehat{H}_2$ deviates by more, nests fall by more, and the income gain shrinks (panels D and E). The endogenous downturn always emerges with knowledge spillovers, and its magnitude increases with ϕ (panels F and G). For large ϕ, a single shock sets off several cycles: lower nest-researcher productivity reduces competition across nests ($NZ^{\eta-\sigma}$ in (45)), pulling research back toward nests between $t \approx 2005$ and 2060 and then toward varieties again.

To map to history, we can interpret the technology shock in Figure 12(A) as the exceptional computing and information-technology breakthroughs that our estimation places in the 1970s and 1980s (Figure 6(B) and Appendix B.2). Under this interpretation, the model associates these computing breakthroughs with the productivity boom of the late 1990s and early 2000s in every case and—when there are positive externalities among nest researchers (basic research)—with the productivity slowdown after 2005, though the model's downturn begins somewhat later than in the data.[38]

[37]This case emerges when we redefine $\bar{H}_2(t)$ in (7) as $\bar{H}_2(t) = \int_{-\infty}^{t} e^{\phi(s-t)} H(s)ds$. These two model cases are not separately identified in the estimation (see Appendix C.4).

[38]The reversals in real income in Figure A7(G) are primarily driven by productivity, and the slowdown starts in 2008 when $\phi = 0.05, 0.04, 0.03$, and between 2009 and 2019 in the other cases. Syverson (2017) surveys the literature. Jorgenson, Ho, and Stiroh (2008), Byrne, Oliner, and Sichel (2013), and Fernald (2015) also associate both the productivity boom (1995–2004) and slowdown (2005-) to information technology. Similar to Gordon (2016), the slowdown in Figure A7(G) arises from a protracted period of fewer breakthroughs with high productivity potential.

7.3 Random aggregate shocks

The estimated parameters imply that the expected number of nests $N^*(t)$ averages only 21 from 1875 to 2015, and that the productivity differences between them are sizable: the 90-10 ratio of $\bar{z}^{1/(\eta-\sigma)}$ (nest market shares) draws is 22. We study here the implications of these estimates for aggregate economic fluctuations. In each simulation, we iterate over paths of $\widehat{H}_1$. For each guess of $\widehat{H}_1$, we simulate random births of nests and productivity according to the Poisson and the log-normal distributions in (40) and (41). We then calculate deviations in the value of research $\widehat{v}$ in (44) and $\widehat{\mathcal{V}}$ in (45) to check the research equilibrium.

For each case of ϕ and H_1^*/H^* in Table 5, we conduct 30 simulations, and summarize the moments for the period 1875–2015 corresponding to our data.[39]

TABLE 5: Economic fluctuations

	std. dev. $\log \widehat{w}(t)$	half-cycle length (years)	avg. corr $\widehat{w}(t)$ & $\widehat{H}_1(t)$
$\widehat{H}_1(t) = 1$	0.024	24	n.a.
Panel A: No research spillover, $\widehat{\widehat{H}}_2 = 1$			
$H_1^*/H^* = 0.6$	0.017	22	0.11
$H_1^*/H^* = 0.6$	0.015	20	0.02
$H_1^*/H^* = 0.7$	0.013	19	0.01
$H_1^*/H^* = 0.8$	0.011	19	0.00
Panel B: Research spillovers with $H_1^*/H^* = 0.5$			
$\phi = 0$	0.020	22	0.38
$\phi = 0.01$	0.022	26	0.50
$\phi = 0.02$	0.025	26	0.54
$\phi = 0.03$	0.027	27	0.59
$\phi = 0.04$	0.025	26	0.56
$\phi = 0.05$	0.023	25	0.46

We simulate the model 30 times for each set of parameters specified in a row. The table summarizes the average standard deviation in real income per capita detrended by the balanced growth path $\log \widehat{w}(t)$, the average time span between local minima and local maxima in real income per capita $\log \widehat{w}(t)$, and the correlation between deviations in real income per capita $\widehat{w}(t)$ and in researchers of varieties $\widehat{H}_1(t)$. In the first row, we impose $\widehat{H}_1 = 1$, and hence research is not in equilibrium. Its purpose is to check whether endogenous fluctuations in research labor exacerbate or attenuate fluctuations.

[39] In each simulation, random shocks start in 1850, since 25 years is the peak innovation in a nest after birth. We allow agents to anticipate these shocks since 1775 and for the economy to converge back to the BGP in 2115. In the periods without shocks (1775–1850 and 2015–2115), we follow the procedure in Section 7.2 above that $\widehat{H}_1$ may affect the measure and productivity of nests through K and $\mathbf{z}$, but there is no randomness. The economy is in the BGP before 1775 and after 2115.

Table 5 summarizes the results. There are no research spillovers in Panel A, $\widehat{H}_2 = 1$, and H_1^*/H^* varies from 0.5 to 0.8. When the BGP share of researchers of varieties H_1^*/H^* is large, economic fluctuations are smaller and cycles are shorter. The standard deviation of $\widehat{w}(t)$ ranges from 0.011 to 0.017, and the half-cycle length is about 20 years. Relative to the case $\widehat{H}_1 = 1$, endogenous research reallocation $\widehat{H}_1 \neq 1$ decreases the volatility of real income. Allowing for spillovers among researchers of nests in panel B increases the volatility of real income $\widehat{w}$ and lengthens the cycles.

For an order of magnitude, we use data on real income per capita in the United States from the Maddison Project.[40] The standard deviation in the smoothed and detrended series is 0.105, and the average time between local maxima and minima is ten years (Appendix Figure A6). Compared to Table 5, cycles in the data are deeper and shorter.

The last column of Table 5 presents the average correlation between real income per capita $\widehat{w}$ and $\widehat{H}_1$. The positive correlation in all cases with research spillovers is consistent with Cohen (2025), who finds that fundamental research increases during recessions, using U.S. Census data. The cyclical behavior of research activity, in which significant technological breakthroughs increase the creation of related varieties, is reminiscent of Schumpeter (1934)'s notion that breakthrough ideas draw a "swarm of entrepreneurs" to producing goods related to that idea. Eventually, innovation possibilities around the original idea are exhausted and tight competition pushes entrepreneurs to search for new breakthroughs.[41]

8. Conclusion

We develop a semi-endogenous growth model in which researchers choose between innovating in existing product types and pioneering entirely new categories—*nests*. Each new nest triggers a predictable cycle: a boom in patenting, establishments, and value added that gradually subsides as newer, more productive nests render the original nest obsolete. This boom-and-bust pattern, visible across 140 years of U.S. patent data, is the organizing empirical fact of the paper.

Estimating the model through maximum likelihood, we find that the bell-shaped evolution of patents within classes fits the data remarkably well, with a median R-squared of 0.80 across 650 patent classes. The most influential patents—identified using the text-based measure of Kelly, Papanikolaou, Seru, and Taddy (2021)—are

[40] As observed by Comin and Gertler (2006), conventional business cycle filters sweep large medium-frequency oscillations into the trend. We detrend the data only from the constant growth rate, like the model simulations, and smooth them with a LOWESS (bandwidth 0.1).

[41] Matsuyama (1999) and Acemoglu (2008) present models of economic growth where some equilibria exhibit cyclical behavior oscillating between periods of technology growth and those of capital deepening. The mechanism to escape competition across nests appears in the static model of Fieler and Harrison (2023).

disproportionately concentrated around the years we identify as nest births, providing an out-of-sample validation of the model's core mechanism. The boom in patents is also associated with a boom in value added, number of workers and establishments in related industries.

Quantitatively, the higher productivity of new breakthrough ideas accounts for 83 percent of U.S. economic growth since the 1870s. Breakthroughs ideas are sufficiently rare and heterogeneous that their arrival may generate aggregate economic fluctuations. The birth of a very productive nest draws research toward applied areas (creation of new varieties) and away from fundamental research (creation of nests), thereby endogenous generating a downturn after the boom.

Understand the propagation of these new, influential breakthrough ideas across space, firm boundaries, and workers' occupational choices are promising paths for future work.

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